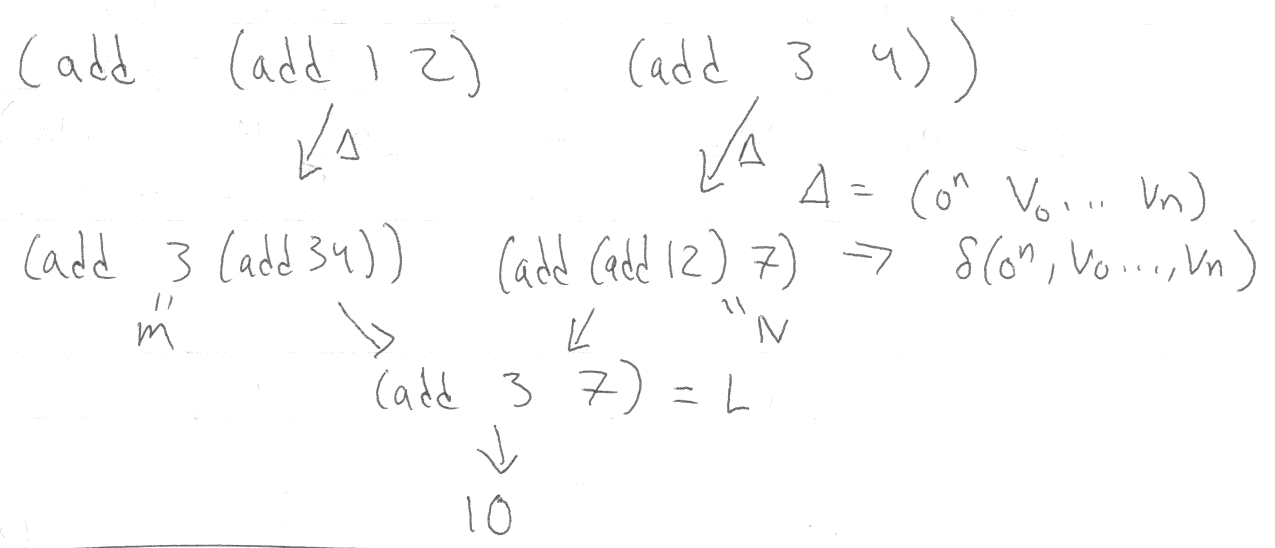
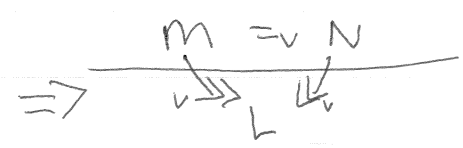


evalv - is partial (errors (stuck terms), divergence)
 $(5 \ 6)$



Consistency:
 If $M \equiv_v N$, $\exists L. M \Rightarrow_v L$ and $N \Rightarrow_v L$



Church-Rosser
 If $K \Rightarrow_v M$ and $K \Rightarrow_v N$, then $\exists L.$
 $M \Rightarrow_v L$ and $N \Rightarrow_v L$

5-2/

~~compatible~~ compatible closure (\rightarrow_v)

"find any work you can"
and do it

\hookrightarrow_v : ISWIM \rightarrow ISWIM

$(o^n v_1 \dots v_n) \hookrightarrow_v \delta(o^n, v_1, \dots, v_n)$ $[\Delta]$

$(\lambda x. m) v \hookrightarrow_v m[x \leftarrow v]$ $[\beta_v]$

$(m N) \hookrightarrow_v (m' N')$

if $m \hookrightarrow_v m'$

and $N \hookrightarrow_v N'$

$(\lambda x. m) \hookrightarrow_v (\lambda x. m')$

if $m \hookrightarrow_v m'$

$(o^n m_1 \dots m_n) \hookrightarrow_v (o^n m'_1 \dots m'_n)$

if $m_i \hookrightarrow_v m'_i$

$(\text{add } (\text{add } 12) (\text{add } 34))$

\hookrightarrow_v

$(\text{add } 3 \quad \quad \quad 7)$

\hookrightarrow_v

10

$\frac{f(x, y, z)}{x=0}$
 $f(++x, ++x, ++x)$

$f(1, 2, 3)$

$f(2, 1, 3)$

$f(3, 1, 2)$

`int main() { return 42; }`

\rightarrow 42

`int f() { f(); }`

↓
↓
↓
↓
↓
↓

$\text{eval}_v(m) = A_0$

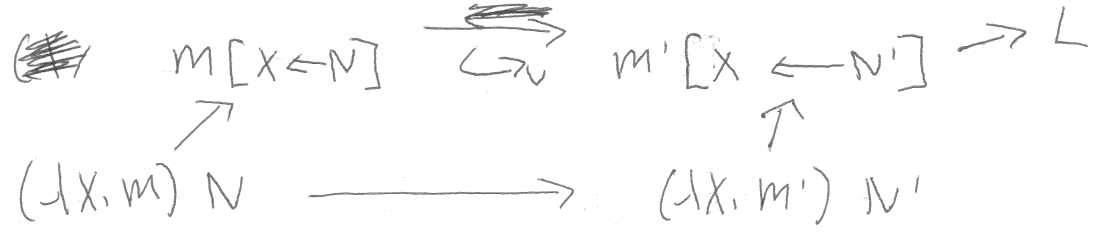
$\wedge \text{eval}_v(m) = A_1$

$\Rightarrow A_0 = A_1$

5-5/

Substitution Lemma:

If $M \hookrightarrow_v M'$ and $N \hookrightarrow_v N'$, then



different!

$(\lambda X. M X) \eta M$

```

int f(int x) {
  return g(x);
}

```

$\Rightarrow g$

$(\lambda X. 5 X) \Rightarrow 5$

Observational Equivalence

- Do two programs do the same thing?

\rightarrow_v does this? } NO. (add 5 10) dtst
 \hookrightarrow_v does this? } ~~(add 5 10)~~
 (+ 7 8)

\equiv_v is \rightarrow_v and \rightarrow_v^R for $(i = 0 \dots 50)$

$x = i * 10$

$a[i] = x$

$(\lambda X. M) A \Rightarrow x = 0, \text{ for } (i = 0 \dots 50)$
 cmp
 $(\lambda Y. N) A \Rightarrow x = x + 10$
 $a[i] = x$

5-4/

A context := a program with a hole

$$(\blacksquare A) = C$$

fill the hole in C with X, C[X]

$$(X A)$$

C =	\blacksquare	$\blacksquare [N] = N$
	(X, C)	(X, C)[N] = (X, C[N])
	(C M)	(C M)[N] = (C[N] M)
	(M C)	(M C)[N] = (M C[N])
	(o^n m ... C M ...)	[N] = (o^n m ... C[N] M ...)

"observational equivalence" of M and N is \approx

$$\forall c. C[M] =_v C[N]$$

(\blacksquare 5)	(\blacksquare +)	(X, Ω) \approx (X, (Ω - Ω))
(\blacksquare 8)	\blacksquare	

(put it in a list, reverse list, get last elem, then call it)

\rightarrow_v compatible closure

$C [(X, M) v]$	\rightarrow_v	$C [M [X \leftarrow v]]$
$C [(o^n v_1, \dots, v_n)]$	\rightarrow_v	$C [\delta(o^n, v_1, \dots, v_n)]$

$$(+ (+ 7 8) (+ 1 z))$$

$$(+ (+ 7 8) \blacksquare) [(+ 1 z)] \rightarrow_v (+ (+ 7 8) \blacksquare) [3]$$

$$(+ (+ 7 8) 3)$$