Typed-ISWIM

$\Delta : b \to B$
$\Delta : o^n \to B^{n+1}$

$M = b$
$\quad | \; o^n \, M \ldots$

$T = B$

$$\frac{}{\vdash b : \Delta(b)}$$

$$\frac{\vdash M_i : T_i \qquad \Delta(o^n) = (T_0 \ldots T_n, T_R)}{\vdash o^n \, M_0 \ldots M_n : T_R}$$

$M = \ldots$
$\quad | \; X$
$\quad | \; \lambda X . M$
$\quad | \; M \; M$

$T = \ldots$
$\quad | \; \underset{\text{dom}}{T} \to \underset{\text{range}}{T}$

$\Gamma = \emptyset \; | \; \Gamma[X \mapsto T]$

$M = \ldots$
$\quad | \; X \text{:} T$
$\quad | \; \lambda (X \text{:} T) . M$

guess (= non-det (= slow))

$$\frac{\Gamma[X \mapsto D] \vdash M : \underbrace{R}}{\Gamma \vdash (\lambda X . M) : \underbrace{D} \to \underbrace{R}}$$

$$\frac{\Gamma(X) = T}{\Gamma \vdash X : T}$$

" $\underset{\text{Input}}{\Gamma} \vdash \underset{\text{Input}}{M} : \underset{\text{Output}}{T}$ " = " Γ proves that M has type T "

$M = \ldots \; | \; \lambda X \text{:} T . M$

$$\frac{\Gamma[X \mapsto T] \vdash M : R}{\Gamma \vdash (\lambda X \text{:} T . M) : T \to R}$$

$$\frac{\Gamma \vdash M : D \to R \qquad \Gamma \vdash N : D}{\Gamma \vdash (M \; N) : R}$$

λ assumes input is valid
λ proves the body is valid

@ proves input is valid
@ assumes body is valid

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$$\Omega = \omega \ \omega \qquad\qquad \Omega_T = \omega_T \ \omega_T$$

$$\omega = \lambda x.\ x \ x \qquad\qquad \omega_T = \lambda x{:}T.\ (x \ x)$$

$$\Omega \to \Omega \qquad\qquad \Omega_T \to \Omega_T$$

works, not stuck, no error

$$\underline{\emptyset \vdash \Omega_T : T_0} \qquad\qquad T_1 \to T_0$$

$$\underline{\emptyset \vdash \omega_T : T_1 \to T_0} \qquad \underline{\emptyset \vdash \omega_T : T_1} \qquad T_1 = T_2 \to T_3$$

$$\underline{\emptyset[x{:}T_1] \vdash (x \ x) : T_0} \qquad \emptyset[x{:}T_1] \vdash (x \ x) : T_0$$

$$\underline{\emptyset[x{:}T_1] \vdash x : T_1 \to T_0} \qquad \underline{\emptyset[x{:}T_1] \vdash x : T_1}$$

$$\emptyset[x{:}T_1](x) = T_1 \to T_0 \qquad \emptyset[x{:}T_1](x) = T_1$$

$$T_1 = T_1 \to T_0$$

If T_1 & T_0 exists, s.t. $T_1 = T_1 \to T_0$

then Ω_T has a type

$$F(x) = x \to T_0 \qquad\qquad T_1 = F(T_1)$$

<u>No T_1 exists</u> for Ω_T or for Y_T

Typed-ISWIM has <u>no</u> infinite loops

has <u>no</u> recursive functions

or dynamic loops

has STRONG normalization

(run out of arrows)

$\in \Sigma_0$ (decidable languages)

Curry-Howard Isomorphism

Props have Proofs

$\updownarrow$ $\qquad\qquad\qquad$ $\updownarrow$

Types have Programs

Typed Rec-ISWIM

$M = \ldots \mid (\text{fix } M)$ $\qquad$ $E = \ldots \mid \text{fix } E$

$E[(\text{fix } (\lambda (X{:}T)\ M))] \mapsto E[M[X \leftarrow (\text{fix } (\lambda (X{:}T)\ M))]]$

$$\frac{\Gamma \vdash M : (T_1 \to T_2) \to (T_1 \to T_2)}{\Gamma \vdash (\text{fix } M) : (T_1 \to T_2)}$$

$$\frac{\dfrac{\emptyset[X{:}T] \vdash X : T}{\emptyset \vdash \cancel{X}\ (\lambda X{:}T,\ X) : (T_1 \to T_2)}}{\emptyset \vdash (\text{fix } (\lambda X{:}T,\ X)) : T} \qquad \forall T.$$

TRC-ISWIM

$M = \ldots \mid \text{if } M\ M\ M$ $\qquad$ $E = \ldots \mid \text{if } E\ M\ M$

$E[\text{if } T\ M\ N] \mapsto E[M]$

$E[\text{if } F\ M\ N] \mapsto E[N]$

$$\frac{\Gamma \vdash M_C : \text{Bool} \qquad \Gamma \vdash M_T : R \qquad \Gamma \vdash M_F : R}{\Gamma \vdash (\text{if } M_C\ M_T\ M_F) : R}$$

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assume X is either a string or a number $\qquad$ X : str or num

(if (string? X) (string-upcase X) (add1 X))

: str → str $\qquad$: num → num

occurrence typing

$T = \ldots \mid T \text{ or } T$ $\qquad$ (union types)

$$\frac{\Gamma \vdash M_C : \text{Bool}, \Gamma_T, \Gamma_F \qquad \Gamma_T \vdash M_T : R_T \qquad \Gamma_F \vdash M_F : R_F}{\Gamma \vdash (\text{if } M_C\ M_T\ M_F) : R_T \text{ or } R_F}$$

~~$\Gamma \vdash M_C : \text{Bool} \qquad \Gamma \vdash M_T : R_T \qquad \Gamma \vdash M_F : R_F$~~

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$$M = \ldots \mid \text{pair } M\ M \mid \text{fst } M \mid \text{snd } M$$
$$T = \ldots \mid M \times M$$
$$E = \ldots \mid \text{pair } E\ M \mid \text{pair } V\ E \mid \text{fst } E \mid \text{snd } E$$
$$V = \ldots \mid \text{pair } V\ V$$

$$E[\text{fst } (\text{pair } V\ U)] \mapsto E[V]$$
$$E[\text{snd } (\text{pair } V\ U)] \mapsto E[U]$$

$$\frac{\Gamma \vdash M : T \times R}{\Gamma \vdash \text{snd } M : R}$$

$$\frac{\Gamma \vdash M : T \qquad \Gamma \vdash N : R}{\Gamma \vdash \text{pair } M\ N : T \times R} \qquad \frac{\Gamma \vdash M : T \times R}{\Gamma \vdash \text{fst } M : T}$$

$$\frac{\models A \qquad \models B}{\models A \wedge B} \qquad \frac{A \models C}{A \wedge B \models C} \qquad \frac{B \models C}{A \wedge B \models C}$$

$$M = \ldots \mid \text{inl } M \mid \text{inR } M \mid \text{match } M\ (\lambda X . M)\ (\lambda X . M)$$
$$T = \ldots \mid M + M$$
$$V = \ldots \mid \text{inl } V \mid \text{inR } V$$
$$E = \ldots \mid \text{inl } E \mid \text{inR } E \mid \text{match } E\ (\lambda X . M)\ (\lambda X . M)$$

$$\boxed{\frac{\models A}{\models A \vee B}} \qquad \frac{\Gamma \vdash M : T}{\Gamma \vdash \text{inl } M : T + R} \qquad \frac{\Gamma \vdash N : R}{\Gamma \vdash \text{inr } N : T + R} \qquad \boxed{\frac{\models B}{\models A \vee B}}$$

$$E[\text{match } (\text{inl } V)\ (\lambda X . M)\ (\lambda Y . N)] \mapsto E[(\lambda X . M)\ V]$$
$$E[\text{match } (\text{inr } U)\ (\lambda X . M)\ (\lambda Y . N)] \mapsto E[(\lambda Y . N)\ U]$$

$$\frac{A \models C \qquad B \models C}{(A \vee B) \models C}$$

$$\frac{\Gamma \vdash O : T + R \qquad \Gamma[X \mapsto T] \vdash M : S \qquad \Gamma[Y \mapsto R] \vdash N : S}{\Gamma \vdash \text{match } O\ (\lambda X . M)\ (\lambda Y . N) : S}$$