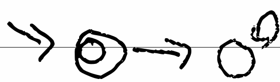
3-1) DFA – $(Q, \Sigma, q_0 \in Q, \delta : Q \times \Sigma \to Q,$
$F \subseteq Q)$

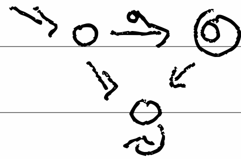
A DFA with no accepts then $L(d) = \emptyset$

$(\{A\}, \{0,1\}, A,$ fun q_i c, ret A,
$\{\})$

A ~~DF~~A acepts only the empty string $L(d) = \{\varepsilon\}$



3-2) $\{a\}$



Char $\rightarrow$ DFA

$\emptyset$ ε $\{a\}$ $\cup$ $\circ$

$\{J\} \circ \{a\} \circ \{y\} \cup \{D\} \circ \{a\} \circ \{y\}$

$\{Jay, Day\}$

3-3)

trace : DFA $\times$ string $\rightarrow$ List (config)

trace d@(Q, Σ, q_0, δ, F) w = c_0 : h d c_0

where c_0 = (q_0, w)

h : DFA $\times$ config $\rightarrow$ List (config)

h d@(Q, Σ, q_0, δ, F) (q_i, w) =

case w of $\varepsilon \rightarrow$ []

cx $\rightarrow$ c_1 : ~~trace~~ h d c_1

where c_1 = (δ(q_i, c), x)

3-4/ Does a DFA accept anything?

example : DFA $\Rightarrow$ False or string

$\to 0 \xrightarrow{a} \xrightarrow{b} \xrightarrow{c} \xrightarrow{d} \xrightarrow{e} ⓞ$ $abcde \in L(D)$

example $(Q, \Sigma, q_0, \delta, F)$ =

$V = \{q_0\}$ $H = \{(q_0, \varepsilon)\}$

while $H \neq \emptyset$

  let $(q_i, w) = H$ first $H = H$.rest

  if $q_i \in F$, ret w

  for $c \in \Sigma$, let $q_j = \delta(q_i, c)$

    if $q_j \notin V$, $V = V \cup \{q_j\}$

      $H = H \cup \{(q_j, wc)\}$

return false

3-5/ complement : DFA → DFA

$L(d) =$ ~~complement~~ $\left(\overline{L(\text{complement}(d))} \right)$

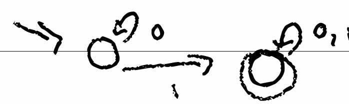
$$(Q, \Sigma, q_0, \delta, F) \Rightarrow (Q', \Sigma', q_0', \delta', F')$$

$$Q' = Q \quad \Sigma' = \Sigma \quad q_0' = q_0 \quad \delta' = \delta$$

$$F' = F^c = Q - F$$

All zeros DFA

not all zeroes



3-6) union : DFA $\times$ DFA $\to$ DFA

$L(\text{union}(A, B)) = L(A) \cup L(B)$

union $(Q_A, \Sigma, q_{0A}, \delta_A, F_A)$ $(Q_B, \Sigma, q_{0B}, \delta_B, F_B)$

$\to (Q_C, \Sigma, q_{0C}, \delta_C, F_C)$

cartesian product

$Q_C = Q_A \times Q_B$

~~[illegible]~~ $(a, b) \in F \times G$

iff $a \in F$

and $b \in G$

$q_{0C} = (q_{0A}, q_{0B})$

$\delta_C((q_a, q_b), c)$

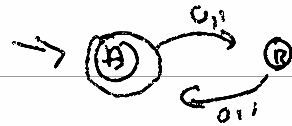
$= (\delta_A(q_a, c), \delta_B(q_b, c))$

$F_C = F_A \times Q_B \cup Q_A \times F_B$

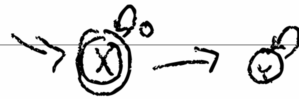
$F_C' = F_A \times F_B$ — intersect

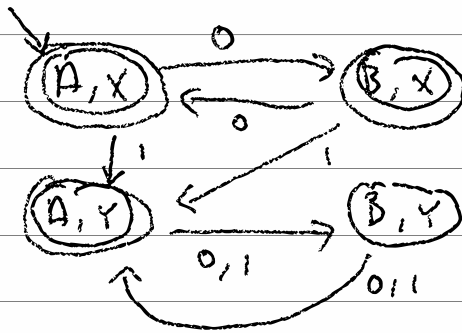
3-7/

even-len



all zeros



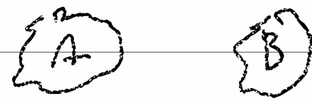


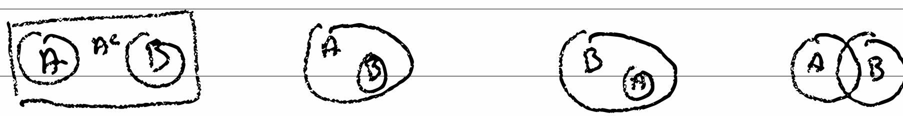
3-8/ Given A and B,

is $L(A) \subseteq L(B)$?

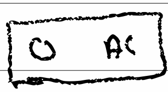
$A \subseteq B$ iff $\forall x \in A.\ x \in B.$

$A \subseteq B$





$B^c \cap A = \emptyset$



subset(A, B) =

example(intersect(complement(B), A)) == false

3.9/ $A = B$ iff $A \subseteq B$ and $B \subseteq A$

model checking and formal verification

$\{Jay, Day\} = J \circ a \circ y \cup D \circ a \circ y$

$\circ : DFA \times DFA \to DFA$