

$$\frac{19-1}{0} \rightarrow 0^a 1 0^b 1 0^{a+b} \rightarrow \cup 0^{a-1} 1 0^b 1 0^{a+b}$$

Diagram showing a transition from a state labeled 0 to a state labeled 0 via a transition labeled 0 . An arrow points from the 0 below the fraction to the 0^a in the expression $0^a 1 0^b 1 0^{a+b}$. Another arrow points from the 0 below the fraction to the 0 in the expression $0^a 1 0^b 1 0^{a+b}$.

$$\downarrow$$
$$0^a 0^b 1 0^{a+b} = 0^{a+b} 1 0^{a+b}$$

\downarrow

$$w \# w$$

$$w \in 0^*$$

$$\# = 1$$

1a-2 / A TM $T = (Q, \Sigma, \Gamma, \delta, q_0, q_a, q_r)$

Q - a finite set of states

Σ - an alphabet

Γ - an alphabet $\Sigma \subset \Gamma$

$w \in \Gamma^* \quad w \in \Sigma^*$

$q_0, q_a, q_r \in Q$ - start, accept, reject

$\delta : \underbrace{Q \times \Gamma}_{\substack{Q - \{q_a, q_r\}}} \rightarrow Q \times \Gamma \times \{L, R\}$

config : $\underbrace{\Gamma^*}_{\text{left}} \times Q \times \underbrace{\Gamma^*}_{\text{right}} = c$

19-3) $w \in L(T)$ iff

$$e[q_0]w \Rightarrow^* x[q_a]y \quad x, y \in \Gamma^*$$

$$\overline{x[q_i]y} \Rightarrow w x[q_i]y w$$

$$\overline{\delta(q_i, a) = (q_j, b, R)}$$

$$\overline{\delta(q_i, a) = (q_j, b, L)}$$

$$x[q_i]ay \Rightarrow x b [q_j]y$$

$$xc [q_j]ay \Rightarrow x [q_i]cby$$

step = config \rightarrow config

$$\text{step } (t, q_i, tr) = (t', q_j, tr')$$

where $(a, y) = \text{look } tr$

$$(q_j, b, d) = \delta(q_i, a)$$

$$(t', tr') = \text{case } d \text{ of } R \rightarrow ((b=t+1), y)$$

$$L \rightarrow (x, c=b-y)$$

look = list char \Rightarrow (char, list(char))

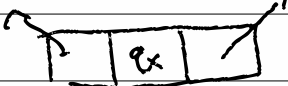
where $(R, x) = \text{look } k-1$

$$\text{look } \epsilon = (w, \epsilon)$$

$$\text{look } (a:x) = (a, x)$$

$$0 \parallel [q_x] \parallel 0$$

$$\epsilon \leftarrow 0 \leftarrow 1 \leftarrow 1 \quad \rightarrow 1 \rightarrow 0 \rightarrow 1 \rightarrow \epsilon$$



19-4 / A computable function f

is a Turing machine.

And

$$f(x) = y$$

iff

$$\exists [q_0] x \Rightarrow^* w [q_a] y$$

add 1

$$0 = 1$$

$$1 = 10$$

$$10 = 11$$

...

$$f(0^x + 0^y) = 0^{x+y}$$

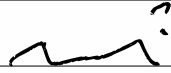
19-5 / when we run a DFA on input w ,
how long could it take to accept
or reject?

$|w|$ steps \Rightarrow yes or
No

when we run a PDA?

$2^{|w|}$ steps \Rightarrow yes or no
or running forever
 \Rightarrow no

19-6 How long does a TM take?



Accept $\exists [q_0]w \Rightarrow \Rightarrow \Rightarrow \Rightarrow \Rightarrow x [q_a]y$



Reject $\exists [q_0]w \Rightarrow \Rightarrow \Rightarrow \Rightarrow \Rightarrow x' [q_r]y'$

Loop $\exists [q_0]w \Rightarrow \Rightarrow \Rightarrow \Rightarrow u [q_i]v \Rightarrow \Rightarrow \Rightarrow \Rightarrow u [q_i]v$

=

Diverging $\forall x, q_i, y, \exists [q_0]w \Rightarrow^* x [q_i]y$

implies $x [q_i]y \Rightarrow x' [q_j]y'$

$\exists x', q_j, y'$ st. $q_j \notin \{q_1, q_2\}$

19-7/ A TM is either

recognizer — may LOOP on
some input

decider — never LOOP

a language A is T-recognizable (Σ_1)
 $\exists M$ recognizer, $L(M) = A$

T-decidable (Σ_0)
 $L(M) = B$

19-81

