M-1

PDAs $\rightarrow$ CFGs

$\leftarrow$

CFG $\rightarrow$ PDA $\qquad \forall g \in CFG.\ \exists p \in PDA.$

$$L(p) = L(g)$$

CFG $\rightarrow$ CFG / CNF $\longrightarrow$ PDA

Chomsky Normal Form

$$R \subseteq P(V \times (V \cup \Sigma)^*)$$

$E \rightarrow 8$

$E \rightarrow E + E$

$E \rightarrow$ if E then E else E

[4-2] In unrestrained CFGs, parse trees are arbitrary arity.

E → 6

E → E + E

E → if E then E else E

number of vars on rhs is the arity of the parse tree node

14-3/ Chomsky Normal Form (CNF)

simplifies context-free grammars to ensure binary trees.

In CNF, every rule is either:

1) $S \to \varepsilon$
2) $A \to BC$ where $A \in V$, B & $C \in V - \{S\}$
3) $A \to a$ where $A \in V$ and $a \in \Sigma$

step 0

14-4

$S \Rightarrow \varepsilon$

$S \Rightarrow 0S1$

step 1: add a new start state

$\Rightarrow$

$S' \Rightarrow S$

$S \Rightarrow \varepsilon$

$S \Rightarrow 0S1$

$\Rightarrow$

$X \Rightarrow AA$

$\Rightarrow$

$X \Rightarrow A$

$X \Rightarrow A$

$X \Rightarrow \varepsilon$

step 2: remove any ε - rules (expect start $\Rightarrow \varepsilon$)

$S' \Rightarrow S$

$S \Rightarrow 0S1$

$S' \Rightarrow \varepsilon$

$S \Rightarrow 01$

step 3: remove "unit" rules $V \Rightarrow U$

$S' \Rightarrow 0S1 \mid 01 \mid \varepsilon$

$S \Rightarrow 0S1 \mid 01$

step 4: add intermediate symbols

$S' \Rightarrow XB \mid AB \mid \varepsilon$

$S \Rightarrow XB \mid AB$

$X \Rightarrow AS$

$A \Rightarrow 0$

$B \Rightarrow 1$

compile a CFG / CNF to a PDA

14-5 in: V, Σ, $R \subseteq P((V \times (V \cup \Sigma)^*))$, $S \in V$

where $r \in R$ is either (S, ε)

(A, BC)

out: Q, Σ, Γ, q_0, δ, F (A, a)

$Q = \{start, loop, end\} \cup V$ $\delta(loop, \varepsilon, \$) \ni \{(end, \varepsilon)\}$

$q_0 = start$ $\delta(start, \varepsilon, \varepsilon) = \{(S, \$)\}$

$\Gamma = V \cup \Sigma \cup \{\$\}$ $\forall A \in V.\ \delta(A, \varepsilon, \varepsilon) = \{(Loop, A)\}$

$F = \{end\}$

if $(S, \varepsilon) \in R$, $\delta(LOOP, \varepsilon, S) \ni (LOOP, \varepsilon)$

$\forall a \in \Sigma$, $\delta(Loop, a, a) \ni (LOOP, \varepsilon)$

if $(A, a) \in R$, $\delta(LOOP, \varepsilon, A) \ni (LOOP, a)$

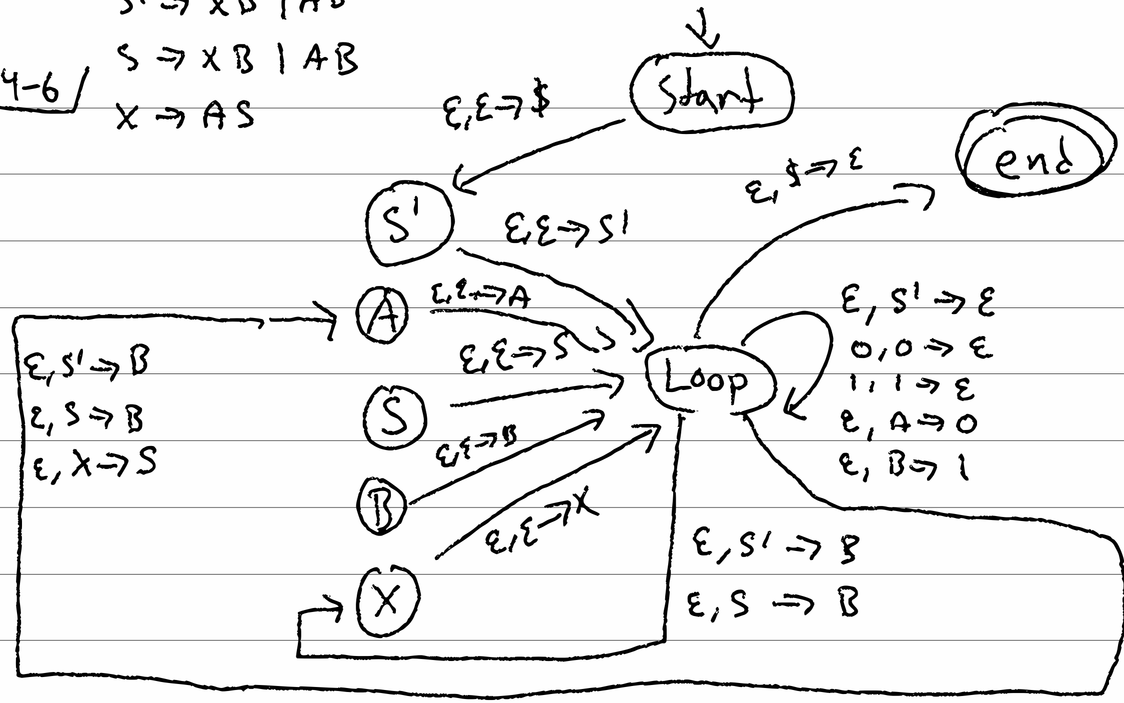
if $(A, BC) \in R$, $\delta(LOOP, \varepsilon, A) \ni (B, C)$

14-6

$S' \to XB \mid AB$

$S \to XB \mid AB$

$X \to AS$



$\varepsilon\,[\text{Start}]\,0011 \to \$\,[S']\,0011 \to \$S'\,[L]\,0011 \to \$B\,[X]\,0011 \to$

$\$BX\,[L]\,0011 \to \$BS\,[A]\,0011 \to \$BSA\,[L]\,0011 \to \$BS0\,[L]\,0011 \to$

$\$BS\,[L]\,011 \to \$BB\,[A]\,011 \to \$BBA\,[L]\,011 \to \$BB0\,[L]\,011 \to \$BB\,[L]\,11 \to$

$\$B1\,[L]\,11 \to \$B\,[L]\,1 \to \$1\,[L]\,1 \to \$\,[L]\,\varepsilon \to \varepsilon\,[\text{end}]\,\varepsilon \to \checkmark$