13-1) Context-free grammars CFG $\Leftrightarrow$ REX

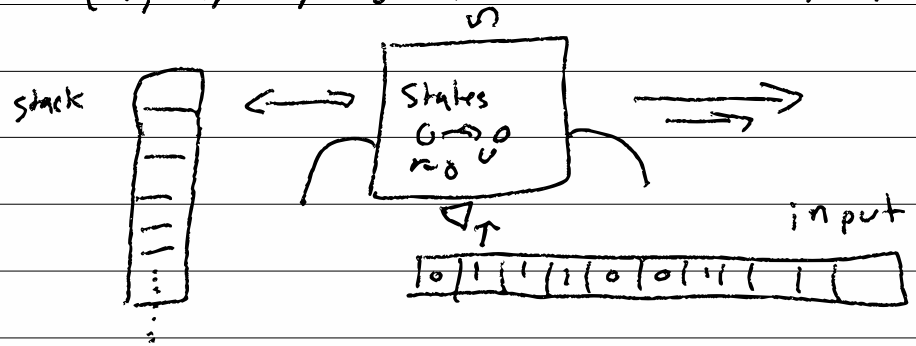
context-free languages CFL $\Leftrightarrow$ REG

push-down automata PDA $\Leftrightarrow$ DFAs

DFAs: $(Q, \Sigma, q_0 \in Q, \delta: Q \times \Sigma \to Q, F \subseteq Q)$

NFAs: $(Q, \Sigma, q_0 \in Q, \delta: Q \times M(\Sigma) \to P(Q), F \subseteq Q)$

PDAs: $(Q, \Sigma, \Gamma, q_0 \in Q, \delta: Q \times M(\Sigma) \times M(\Gamma) \to P(Q \times M(\Gamma)), F \subseteq Q)$



13-2) $0^n 1^n$

0011

ε [A] 0011

\$ [B] 0011

\$ ⏑ [B] 011

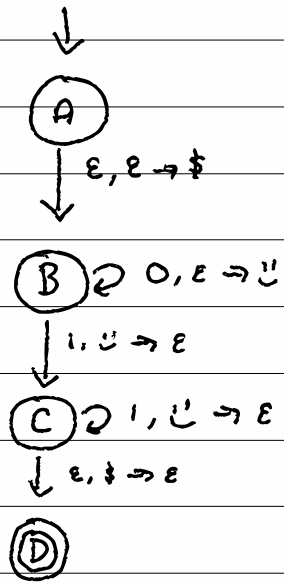
\$ ⏑⏑ [B] 11

\$ ⏑ [C] 1

\$ [C] ε

ε [D] ε

✓



a, b → c

on input a, with stack

b, push c onto stack

$\Sigma = \{0, 1\}$

$\Gamma = \{\$, ⏑\}$

ε[A]011 → \$[B]011 →

\$⏑[B]11 → \$[C]1 →

ε[D]1 → X

ε[A]001 → \$[B]001 →

\$⏑[B]01 → \$⏑⏑[B]1 →

\$⏑[C]ε → X

13-3/ PDA: Q - finite set of states

P Σ - an alphabet for input

Γ - an alphabet for stack

$q_0 \in Q$ — start state

$\delta : Q \times M(\Sigma) \times M(\Gamma) \to P(Q \times M(\Gamma))$

$F \subseteq Q$ — accept states

$w \in L(P)$ iff $\varepsilon[q_0]w \to^* x[q_f]\varepsilon$ s.t. $q_f \in F$

$w \in \Sigma^*$, $x \in \Gamma^*$

$$\frac{\delta(q_i, c, \alpha) \ni (q_j, \gamma)}{\beta\alpha\,[q_i]\,cw \to \beta\gamma\,[q_j]\,w}$$

$\beta \in \Gamma^*$, $\alpha \in M(\Gamma)$

$q_i \in Q$, $c \in M(\Sigma)$

$w \in \Sigma^*$, $\gamma \in M(\Gamma)$

$q_j \in Q$

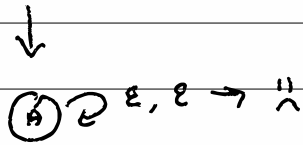
13-4/ Goal: PDAs to represent an algorithm for accepting CFLs

PDAs $\Longleftrightarrow$ CFGs

① $\forall p \in PDA.\ \exists g \in CFG.\ L(p) = L(g)$

② $\forall g \in CFG.\ \exists p \in PDA.\ L(g) = L(p)$

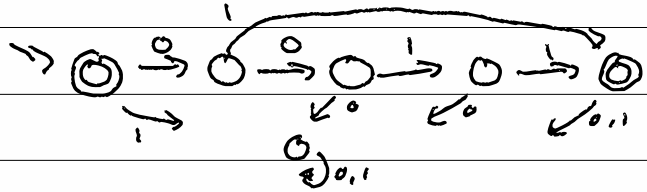
13-5/



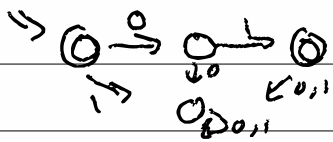
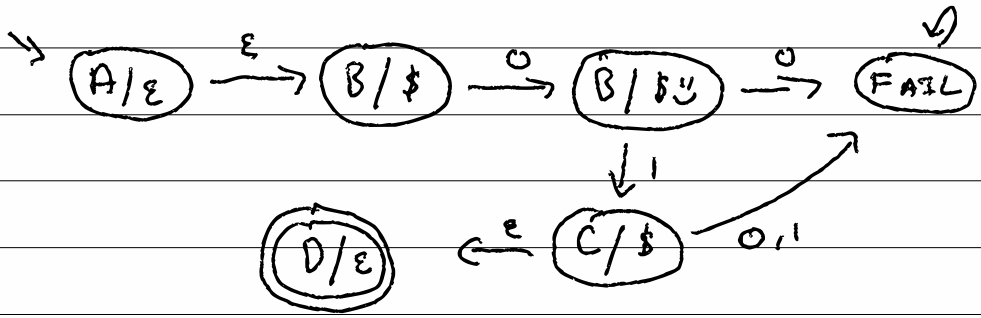
$0^n 1^n$ for a given k.

$\{0^i 1^i \mid \text{s.t. } i \leq k\}$ (in) a DFA (with) REG

k = 2



k = 1



13-6/ $k=1$ is a PDA with a stack limited to size 2