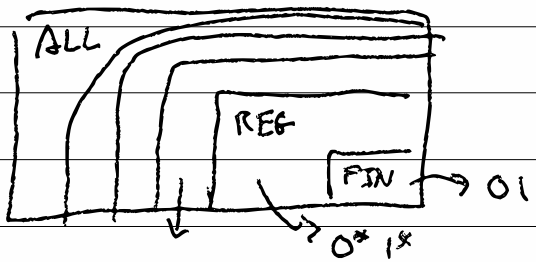
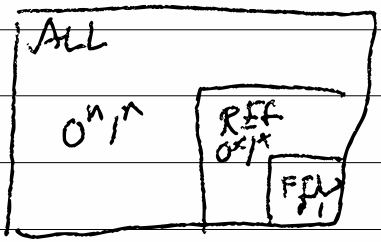
12-1

$0^n 1^n \in ALL$ & REG — can't the regular pumping property





12-2 / CFG - context-free grammar

$0^n 1^n$:

$S \to \varepsilon$ ← rule, transitions

$S \to 0S1$ ← productions

start symbol

variable non-terminal symbol

a string of terminals & nonterminals

$G = (V, \Sigma, R, S)$ $\quad R \subseteq P(V \times (V \cup \Sigma)^*)$

V: variables, some finite set

Σ: alphabet, some finite set

$S \in V$ a start symbol

A B

12-3/ $S \Rightarrow \varepsilon \mid 0S1$

ε B B B A

000111 $S \Rightarrow 0S1 \Rightarrow 00S11 \Rightarrow 000S111 \Rightarrow 000111$

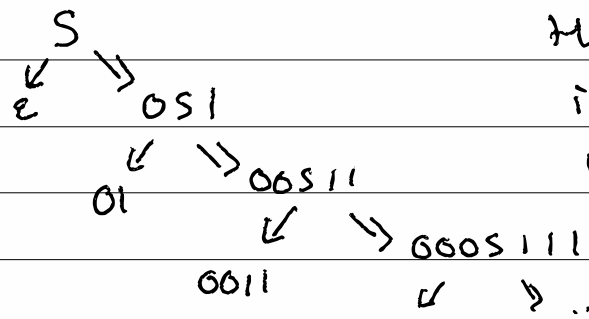
$w \in L(G)$ iff $S \Rightarrow^* w$

$\dfrac{}{w \Rightarrow^* w}$ refl $\qquad \dfrac{w \Rightarrow u \quad u \Rightarrow^* v}{w \Rightarrow^* v}$ trans

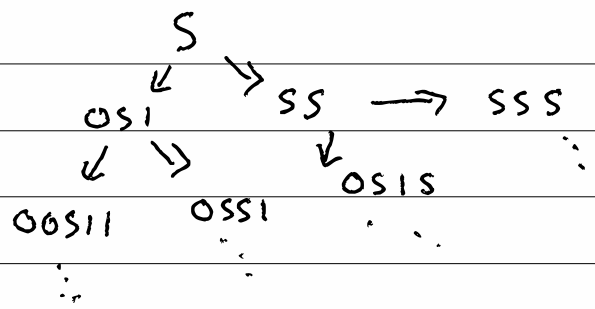
$\dfrac{(A, w) \in R}{uAv \Rightarrow uwv}$ where $u \in \Sigma^* \quad w, v \in (V \cup \Sigma)^*$, $A \in V$

12-4/ $S \to \varepsilon \mid 0S1$

a string is in the language iff it is one of the leaves of the derivation tree



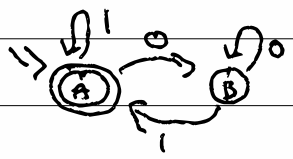
12-5/ $S \to 0S1 \mid SS$



no leaves!

12-6/ DFAs ⊂ ??? ⊂ CFGs

$\not\subseteq$ ⊂ $\{0^n 1^n\}$



F ← +1

$A \to \varepsilon \mid 1A \mid 0B$

$B \to 0B \mid 1A$

Σ J

G G H I J F

$A \to 1A \to 11A \to 110B \to 1100B \to 11001A \to 11001$

12-7/ in: $(Q, \Sigma, q_0, \delta: Q \times \Sigma \to Q, F)$

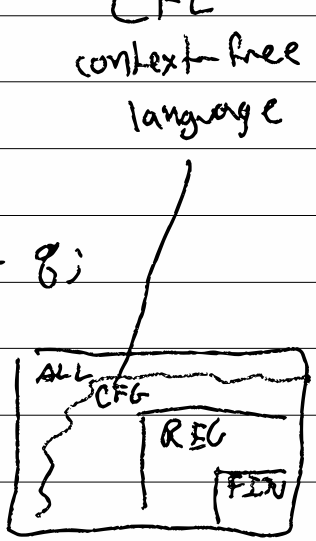
out: (V, Σ, R, S)

$V = Q \qquad q_0 = S$

$q_f \to \varepsilon \quad$ if $\quad q_f \in F$

$q_i \to a\, q_j \quad$ if $\quad \delta(q_i, a) = q_j$

DFA $\subseteq$ CFG



12-8/ $A \cup B$? $S \Rightarrow A.S \mid B.S$

$A \circ B \Rightarrow$ $S \Rightarrow A.S\ B.S$

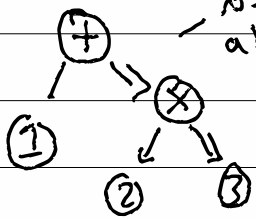
$A \cap B$? not possible

A B C D E R

12-9/ $E \to 0 \mid 1 \mid 2 \mid 3 \mid E+E \mid E \times E$

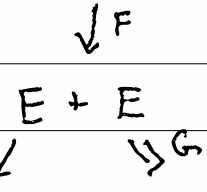
C

$1 + 2 \times 3$



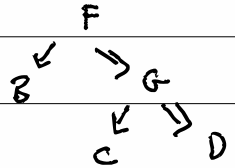
AST abstract syntax tree

E

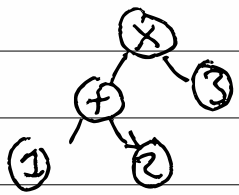


FBGCD

parse tree



$E \overset{G}{\Rightarrow} E \times E \overset{F}{\Rightarrow} E + E \times E \overset{B}{\Rightarrow} 1 + E \times E \overset{C}{\Rightarrow} 1 + 2 \times E \overset{D}{\Rightarrow} 1 + 2 \times 3$



ambiguous