

26-1/

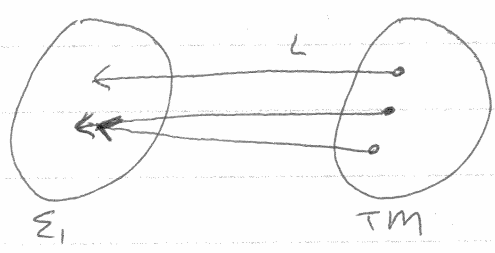
$N \cong \text{Evens}, \mathbb{Q}, \mathbb{Z}, N^2, N^k, \text{TMs}$

$N \not\cong \text{Reals } (\mathbb{R}), [0, 1), \text{IBS}$

Goal: $\Sigma_1 \not\cong \text{ALL}$
 \cong
 $N \not\cong \text{REALS}$

\cong is an equivalence
 $A \cong A$
 $X \cong Y$ then $Y \cong X$
 $X \cong Y$ and $Y \cong Z$ then $X \cong Z$

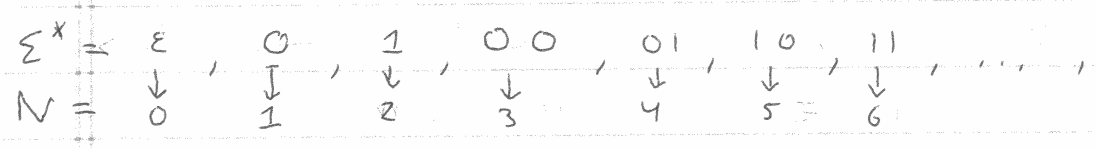
$\Sigma_1 \cong N$? ~~iff~~ $A \in \Sigma_1$ iff $\exists m \in \text{TM}, L(m) = A$



$\Sigma^* \cong N$

Goal: $\text{ALL} \cong \mathbb{R} \cong \text{IBS}$

What is ALL? $A \in \text{ALL}$ iff A is a set of strings of Σ^*
 $\text{ALL} = \mathcal{P}(\Sigma^*)$



$\text{ALL} = \mathcal{P}(\Sigma^*) = \{ \emptyset, \{ \epsilon \}, \{ 0 \}, \{ 1 \}, \dots, \{ 0, 1 \}, \{ 0, 00 \}, \dots$

elements of ALL ($\mathcal{P}(\Sigma^*)$) are sets of strings
 member: $\underbrace{\text{Elem}}_{\Sigma^*} \Rightarrow \underbrace{Y/N}_{0 \text{ or } 1}$ \hookrightarrow membership function
 $(\lambda n. n \% 2 == 0) =$ ~~evens~~ ^{something weird}

elements of ALL are IBSes

$\text{IBS} = \lambda n. 0 = \emptyset$

26-2/ We know: $\overline{A_{TM}} \notin \Sigma_1$
 and infinitely many other things $\notin \Sigma_1$

Reducibility

$A \stackrel{\in ALL}{\leq_m} B \stackrel{\in ALL}{} (A \leq_m B)$ if $\exists f \in \text{computable fun}$
 $f: \Sigma^* \rightarrow \Sigma^*$ (Turing Machine outputs answer as output)

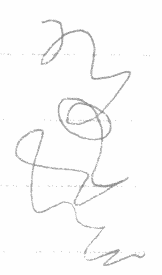
Sit. ~~is~~

$$\forall w \in \Sigma^*, f(w) \in B \text{ iff } w \in A$$

If you have a "B" machine and an "f" machine, then you can make an "A" machine.

$$A \leq_m B \text{ and } B \in \Sigma_0 \Rightarrow A \in \Sigma_0$$

$$A \leq_m B \text{ and } A \notin \Sigma_0 \Rightarrow B \notin \Sigma_0$$



$$E_{TM} = \{ \langle M \rangle \mid M \in TM \text{ and } L(M) = \emptyset \} \notin \Sigma_0$$

$A_{TM} \leq_m E_{TM}$ via a way to turn A_{TM} problems into E_{TM} problems

$$f(\langle M, w \rangle) \in E_{TM} \text{ if } \langle M, w \rangle \in A_{TM}$$

$$L(f(\langle M, w \rangle)) = \emptyset \quad f: \langle M, w \rangle \rightarrow \langle M' \rangle$$

$$\cancel{L(M')} \quad L(M') = \emptyset \text{ iff } w \in L(M)$$

M' = "On input x , simulate M on w
 if accepts, reject
 o.w., accept"

$$\nexists LL_{TM} \notin \Sigma_0$$

Handwritten text at the top of the page, possibly a title or header.

Handwritten text, possibly a date or a specific reference.

Handwritten text, possibly a name or a subject.

Handwritten text, possibly a list or a set of instructions.

Handwritten text, possibly a list or a set of instructions.

Handwritten text, possibly a name or a subject.

Handwritten text, possibly a name or a subject.

Handwritten text, possibly a name or a subject.

Handwritten text, possibly a name or a subject.

Handwritten text, possibly a name or a subject.

Handwritten text, possibly a name or a subject.

Handwritten text, possibly a name or a subject.

Handwritten text, possibly a name or a subject.

Handwritten text, possibly a name or a subject.

26-3/

$$\text{REG}_{\text{TM}} = \{ \langle M \rangle \mid M \in \text{TM} \text{ and } L(M) \in \text{REG} \}$$
$$\in \Sigma_0$$

$$f: \langle M, w \rangle \rightarrow \langle M' \rangle$$

$$\text{ATM} \leq_m \text{REG}_{\text{TM}}$$

$$f(\langle M, w \rangle) \in \text{REG}_{\text{TM}} \text{ iff } \langle M, w \rangle \in \text{ATM}$$

$$L(M') \in \text{REG} \text{ iff } w \in L(M)$$

M' = "On input x ,
simulate M on w
if accepts, then accept
o.w. check if x is $0^n 1^n$ "

$\text{CFL}_{\text{TM}} \in \Sigma_0$ by picking $0^n 1^n 0^n$

Rice's Theorem proves all "non-trivial" P are undecidable

$$\text{EQ}_{\text{TM}} = \{ \langle M, N \rangle \mid L(M) = L(N) \}$$

$$\text{ETM} \leq_m \text{EQ}_{\text{TM}}$$

$$f(\langle M \rangle) = \langle M, \emptyset \rangle$$

Variant of TM called LBA
Linear Bounded Automata

$LBA = (Q, \Sigma, \Gamma, q_0, \delta: Q \times \Gamma \rightarrow Q \times \Gamma \times \{L, R\}, q_a, q_r)$
 $w \in L(L) \iff w [q_0] w \Rightarrow^* u [q_a] v$

but there is no rule for adding blanks

~~LBA~~ = TM w/ a finite tape (input + 2 blanks)

~~TM: $u [q_i] v \Rightarrow w [q_i] v w$~~
TM: $u [q_i] v \Rightarrow w [q_i] v w$

LBA: $\delta(q_i, a) = (q_j, b, R) \quad \delta(q_i, a) = (q_j, b, L)$
 $w [u [q_i] a] v w \Rightarrow w [u [q_j] b] v w \quad w [u [q_i] a] v \Rightarrow w [u [q_j] c] b v$

ALL examples of Σ_0 were really LBAs

$\Gamma = \Sigma \cup \{\sqcup\}$ useful $\Gamma_n = \Gamma \times \mathbb{Z}$
 $\Gamma^2 = \Sigma \cup \{\sqcup\} \cup (\Gamma_n \times \Gamma_n)$

ALBA $\in \Sigma_0$ LBAs — accept
reject
diverge $\left\{ \begin{array}{l} \text{loop} \\ \text{spin} \end{array} \right.$

config = $u [q_i] v = Q \times N \times \Gamma^* \times \Gamma^*$
 $\downarrow \quad \downarrow \quad \downarrow \quad \downarrow$
 $Q \times |w| \times |\Gamma|^{|w|} \times |\Gamma|^{|w|}$

ELBA

ATM Σ_m ELBA

make an LBA to check if an M run accepts w
if no valid runs exist, then $w \notin L(M)$

1. $\frac{1}{x^2} = x^{-2}$

2. $\frac{1}{x^3} = x^{-3}$

3. $\frac{1}{x^4} = x^{-4}$

4. $\frac{1}{x^5} = x^{-5}$

5. $\frac{1}{x^6} = x^{-6}$

6. $\frac{1}{x^7} = x^{-7}$

7. $\frac{1}{x^8} = x^{-8}$

8. $\frac{1}{x^9} = x^{-9}$

9. $\frac{1}{x^{10}} = x^{-10}$

10. $\frac{1}{x^{11}} = x^{-11}$

11. $\frac{1}{x^{12}} = x^{-12}$

12. $\frac{1}{x^{13}} = x^{-13}$

13. $\frac{1}{x^{14}} = x^{-14}$

14. $\frac{1}{x^{15}} = x^{-15}$

15. $\frac{1}{x^{16}} = x^{-16}$

16. $\frac{1}{x^{17}} = x^{-17}$

17. $\frac{1}{x^{18}} = x^{-18}$

18. $\frac{1}{x^{19}} = x^{-19}$

19. $\frac{1}{x^{20}} = x^{-20}$

20. $\frac{1}{x^{21}} = x^{-21}$

21. $\frac{1}{x^{22}} = x^{-22}$