$N \simeq$ Evens, $\mathbb{Q}$, $\mathbb{Z}$, N^2, N^k, TMs

$N \not\simeq$ Reals ($\mathbb{R}$), $[0,1)$, IBS

Goal: $\Sigma_1 \not\simeq$ ALL

$\simeq$ is an equivalence

$A \simeq A$

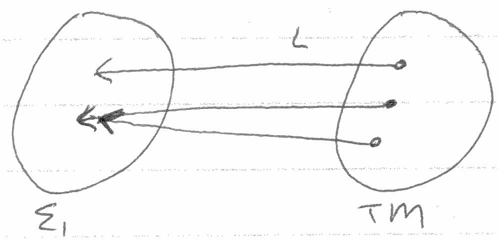
$X \simeq Y$ then $Y \simeq X$

$X \simeq Y$ and $Y \simeq Z$ then $X \simeq Z$

$$\underset{\simeq}{\Sigma_1} \not\simeq \underset{\simeq}{\text{ALL}}$$

$N \not\simeq$ REALS

$\Sigma_1 \simeq N$? $A \in \Sigma_1$ iff $\exists m \in TM$, $L(m) = A$



$\Sigma^* \simeq N$

Goal: ALL $\simeq$ ~~RE~~ $\simeq$ IBS

What is ALL? $A \in$ ALL if A is a set of strings of Σ^*

ALL $= \mathcal{P}(\Sigma^*)$

$$\begin{array}{cccccccccccccccc} \Sigma^* = & \varepsilon, & 0, & 1, & 00, & 01, & 10, & 11, & \ldots, \\ & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \\ N = & 0 & 1 & 2 & 3 & 4 & 5 & 6 & \end{array}$$

ALL $= P(\Sigma^*) = \emptyset \quad \{\varepsilon\} \quad \{0\} \quad \{1\} \quad \ldots \quad \{0,1\}, \{0, 000\} \ldots$

elements of ALL ($P(\Sigma^*)$) are $\underbrace{\text{sets}}$ of strings

↳ membership function

member : $\underbrace{\underbrace{\text{Elem}}_{\Sigma^*}}_{N} \Rightarrow \underbrace{Y/N}_{0 \text{ or } 1}$

$(\lambda n.\ n \% 2 == 0) =$ ~~Evens~~ something weird

elements of ALL are IBSes

IBS $= \lambda n.\ 0 = \emptyset$

26-2/ We know: $\overline{A_{TM}} \notin \Sigma_1$

and infinitely many other things $\notin \Sigma_1$

Reducibility

$A^{\in ALL}$ is reducible to $B^{\in ALL}$ ($A \leq_m B$) if $\exists f \in$ computable fun (Turing that puts answer as output)

$f : \Sigma^* \Rightarrow \Sigma^*$

s.t. $\forall w \in \Sigma^*$, $f(w) \in B$ iff $w \in A$

If you have a "B" machine and an "f" machine, then you can make an "A" machine.

$A \leq_m B$ and $B \in \Sigma_0 \Rightarrow A \in \Sigma_0$

$A \leq_m B$ and $A \notin \Sigma_0 \Rightarrow B \notin \Sigma_0$

$E_{TM} = \{ \langle M \rangle \mid M \in TM \text{ and } L(M) = \emptyset \} \notin \Sigma_0$

$A_{TM} \leq_m E_{TM}$ via a way to turn A_{TM} problems into E_{TM} problems

$f(\langle M, w \rangle) \in E_{TM}$ iff $\langle M, w \rangle \in A_{TM}$

$L(f(\langle M, w \rangle)) = \emptyset$ $\quad f : \langle M, w \rangle \longrightarrow \langle M' \rangle$

$L(M') = \emptyset$ iff $w \in L(M)$

M' = "On input x, simulate M on w
if accepts, reject
o.w., accept"

$ALL_{TM} \notin \Sigma_0$

26-3

$REG_{TM} = \{ <M> \mid M \in TM \text{ and } L(M) \in REG \}$

$\notin \Sigma_0$

$f: <M,w> \to <M'>$

$A_{TM} \leq_m REG_{TM}$

$f(<M,w>) \in REG_{TM}$ iff $<M,w> \in A_{TM}$

$L(M') \in REG$ iff $= w \in L(M)$

M' = "On input x,
simulate M on w
if accepts, then accept
o.w. check if x is $0^n 1^n$ "

$CFL_{TM} \notin \Sigma_0$ by picking $0^n 1^n 0^n$

Rice's Theorem proves all "non-trivial" P are undecidable

$EQ_{TM} = \{ <M,N> \mid L(M) = L(N) \}$

$E_{TM} \leq_m EQ_{TM}$

$f(<M>) = <M, \emptyset>$

26-4

Variant of TM called LBA

Linear Bounded Automata

$LBA_\ell = (Q, \Sigma, \Gamma, q_0, \delta : Q \times \Gamma \to Q \times \Gamma \times \{L, R\}, q_a, q_r)$

$w \in L(\ell)$ iff $\sqcup [q_0] w \sqcup \Rightarrow^* u [q_a] v$

but there is no rule for adding blanks

~~LBA~~ = TM w/ a finite tape (input + 2 blanks)

$$\text{TM:} \quad \frac{}{u[q_i]v \Rightarrow \sqcup u [q_i] v \sqcup}$$

$$\text{LBA:} \quad \frac{\delta(q_i, a) = (q_j, b, R)}{\sqcup u [q_i] a v \sqcup \Rightarrow \sqcup u b [q_j] v \sqcup} \qquad \frac{\delta(q_i, a) = (q_j, b, L)}{u c [q_i] a v \Rightarrow u [q_j] c b v}$$

ALL examples of Σ_0 were really LBAs

$\Gamma = \Sigma \cup \{\sqcup\} \cup$ useful $\qquad \Gamma_\wedge = \Gamma \times 2$

$\Gamma^2 = \Sigma \cup \{\sqcup\} \cup (\Gamma_\wedge \times \Gamma_\wedge)$

$A_{LBA} \in \Sigma_0$ LBAs — accept

reject

diverge < loop, spin

config $= u [q_i] v = Q \times N \times \Gamma^* \times \Gamma^*$

$\to Q \times |w| \times |\Gamma|^{|w|}$

E_{LBA}

$A_{TM} \leq_m E_{LBA}$ make an LBA to check if an M run accepts w

if no valid runs exist, then $w \notin L(M)$