Theory of Computation

What is "computation"?

- ↳ not about software / hardware / "algorithsms"
- ↳ logic, philosophy, truth
- ↳ but in <u>math</u>

A computer = A problem = A set

{ "$x + y = z$" where $x+y$ actually is z }

"$1+1=2$" $\in$ "$1+1=3$" $\notin$

Thy of Computation — a theory of defining sets

A set is a bunch of stuff) — the universe — $\mathcal{U}$

some in / some out

{chair, squi, bulb} {a, b, c} {1, 2}

Finite Set — could write them down

Infinite Set — can't write all the stuff in them

$x \in A$ means A is a set and x is in it.

$\emptyset$: $(\forall x.\ x \notin \emptyset)$

$X \subset Y$: $(\forall a.\ a \in X \rightarrow a \in Y)$

$X \cup Y$: $a \in X \cup Y$ iff $a \in X$ or $a \in Y$

$X \cap Y$: $a \in X \cap Y$ iff $a \in X$ and $a \in Y$

$X^c, \bar{X}$: $a \in X^c$ iff $a \notin X$ (implicitly $a \in \mathcal{U}$)

$P(X)$: $a \in P(X)$ iff $a \subset X$

$\mathbb{N}$ — naturals $0 \in \mathbb{N}$ $2 \in \mathbb{N}$

$\mathbb{Z}$ — integers $-1 \in \mathbb{Z}$ $5 \in \mathbb{Z}$

$\mathbb{R}$ — reals $\pi \in \mathbb{R}$ $2 \in \mathbb{R}$ $\sqrt{2} \in \mathbb{R}$

$\mathbb{Q}$ — rationals $1/2 \in \mathbb{Q}$ $3/4 \in \mathbb{Q}$

1-2

Finite computers are finite sets and you can write them down!

pairs (a, b) $X \times Y$ $A \times B$ cross product / cartesian product

= a set of pairs where the left is from A and right from B

tuples – n-pair

pair = 2-tuple

$(a, b) \in A \times B$ iff $a \in A$ and $b \in B$

$(a, b, c) \in A \times B \times C$ iff $a \in A$, $b \in B$, and $c \in C$

Relation is a set of tuples $(A \times B)$ $(E \times F \times G)$

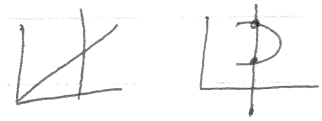
interesting : $(A \times A)$ $a R b$ to mean $(a, b) \in R$

reflexive : $\forall a.\ a R a$

symmetric : $\forall a, b.\ a R b \rightarrow b R a$

trans : $\forall a, b, c.\ a R b \wedge b R c \rightarrow a R c$

equiv : refl + trans + sym



Function is a relation s.t.

$a R b$ and $a R c \rightarrow b = c$

$f(a) = b$ $f(a) = c$

$a R b$

domain of fun is the left of rel

range right

$f : A \rightarrow B$ A = domain

$f \subset A \times B$ B = range