Regular Languages (REG)

- DFAs — NFAs — REX (regular expressions)

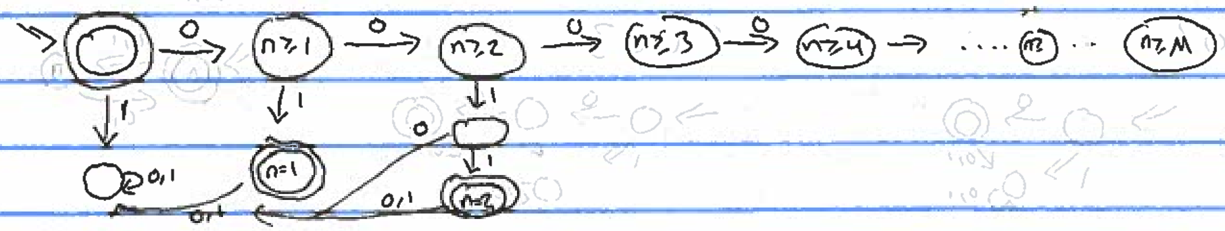
NFA $\Leftrightarrow$ DFA REX $\Leftrightarrow$ NFA

B $w \in B$ iff $w = 0^n 1^n$ for some $n \in \mathbb{N}$

$\varepsilon \in B$ $(n=0)$ $01 \in B$ $(n=1)$ $0011 \in B$ $(n=2)$

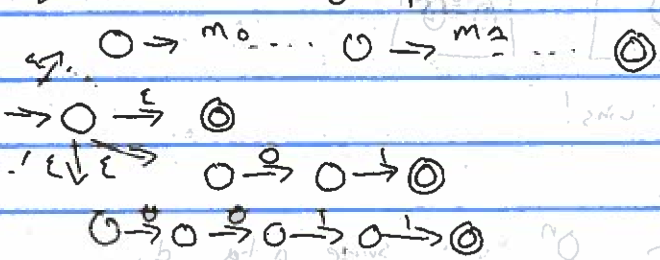
$B \neq 0^* 1^*$ $001 \in 0^* 1^*$ $001 \notin B$ $n \geq 5{,}169{,}342$

$M = ??$



C_m $w \in C_m$ iff $w = 0^n 1^n$ for some $n \leq M$

$B \neq C_m$ $0^{m+1} 1^{m+1} \in B$ $0^{m+1} 1^{m+1} \notin C_m$



What does this say about B? ALL $= P(\Sigma^*)$

1. B is illegal in some way $\Leftrightarrow$ $B \notin$ ALL
2. B is just not in REG $\Leftrightarrow$ $B \in$ ALL, $B \notin$ REG

$x \notin$ REG $\Leftrightarrow$ $\forall d \in$ DFA, $L(d) \neq x$ $d_0, d_1, d_2 \neq B$

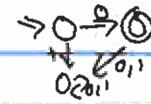
① Property P s.t. $\forall r \in$ REG, $P(r)$

② $\neg P(B)$

① + ② $\Rightarrow$ $B \notin$ REG

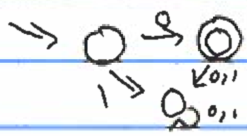
8-2 P = some property true of all regular languages : ~~REG~~ Language → Prop

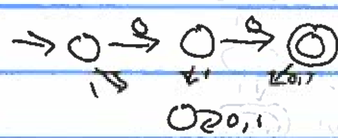
$P_1(d)$ = d has a start state X P_1 : DFA → Prop

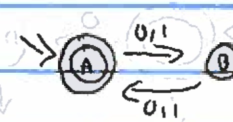
$P_2(r)$ = $\varepsilon \in r$ X  Isn't true on all REGs

what's the smallest DFA for $\emptyset$? what smallest $\{\varepsilon\}$

$\{0\}$ 

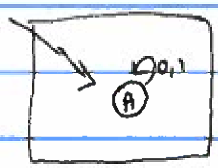
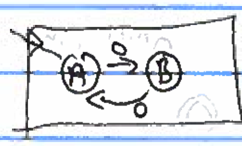
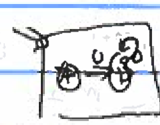
$\{00\}$ 

Even 

$w = 00$ $w \in L(\cdot)$
$|w| \geq |Q|$ $2 \geq 2$
A B A

All DFAs have loops Suppose $|Q| = M$

$0^{M+1} \in L(d)$?

$|Q| = 1$ Jay wins Jay wins!

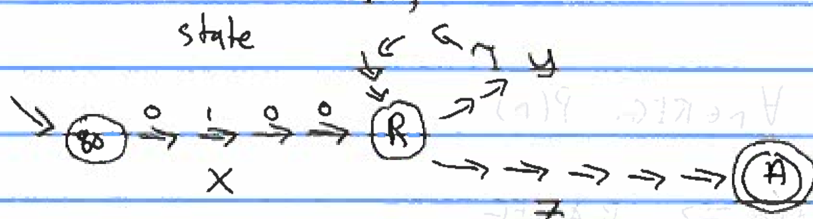
How many states do we visit on 0^n for some dfa d?

[1,] min(|Q|, n+1)


$x = \varepsilon$, $z = \varepsilon$

whenever $n \geq |Q|$, then some state repeats $w = 000$ $y = 000$

Assume: $w \in L(d)$ and $|w| \geq |Q|$
we end in an accepting state we hit a loop ⟹



$w = xyz$ $[q_0]\,x \Rightarrow^* [R]$
x could be ε $[R]\,y \Rightarrow^* [R]$
z could be ε $[R]\,z \Rightarrow^* [A]$

$x = 0100$

$xz \in L(d)$ $xyyyz \in L(d)$ $\forall i \in \mathbb{N}, xy^iz \in L(d)$

3-3

$P_{DFA}(d) =$

$\forall (w \in \underbrace{L(d)} \mid |w| \geq \underbrace{|Q|})$

$\exists (x, y, z \in \Sigma^* \mid w = x \circ y \circ z \wedge |y| > 0$

$|xy| < \underbrace{|Q|})$

$\forall i \in N,$

$xy^iz \in \underbrace{L(d)}$

$P(A) =$ "Regular Pumping Property" RPP

① $\exists p \in N.$

$\forall (w \in A \mid |w| \geq p)$

$\exists (x, y, z \in \Sigma^* \mid w = xyz \wedge |y| > 0 \wedge |xy| < p)$

$\forall i \in N,$

$xy^iz \in A$

$\neg \forall P(x) \Longleftrightarrow \exists \neg P(x)$

② $\neg P(B) ::=$

$\neg \exists P(x) \Longleftrightarrow \forall \neg P(x)$

$\forall p \in N.$

$\exists (w \in B \mid |w| \geq p)$

$\forall (x, y, z \in \Sigma^* \mid w = xyz \wedge |y| > 0 \wedge |xy| < p)$

$\exists i \in N$

$xy^iz \notin B$

$B \notin REG$

$\Rightarrow REG \neq ALL$

given: p choose: $w = 0^p 1^p$

given: x, y, z $\quad 0^p1^p = xyz \quad |y| \geq 0 \quad |xy| < p$

$x = 0^a \quad y = 0^b \quad z = 0^c 1^p \quad b \geq 0 \quad a + b < p$

$a + b + c = p$

$xy^iz = 0^a 0^{bi} 0^c 1^p = 0^{a+bi+c} 1^p \in B$

iff $a + bi + c = p$

$a + b + c = a + bi + c \Rightarrow b = bi \Rightarrow i = 1$

choose: $i = 0$

8-4