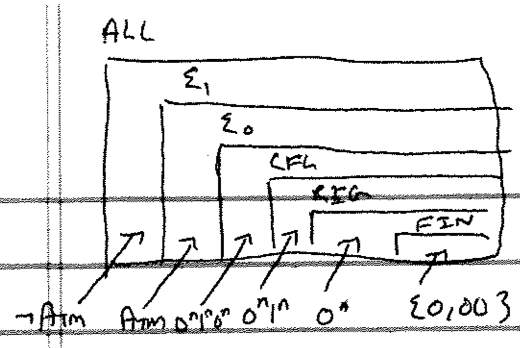

$ALL \not\approx R$

$\Sigma_1 \not\approx N \qquad \Sigma_0 \approx N$

$\Sigma_1 \neq \Sigma_0$

If $A \notin \Sigma_0$, then $\overline{A} \notin \Sigma_1$ (or $A \notin \Sigma_1$)

Reducibility

$\underbrace{A \leq_m}_{\text{is reducibile}} B$ if $\exists$ comp func. f where $\forall w \in \Sigma^*$.

$w \in A$ iff $f(w) \in B$

ex. A = decmath $\quad B$ = bin math

"$4+1=5$" $\in A \quad$ "$100+1 = 101$"

f = dec to binary conversion

If $A \leq_m B \wedge B \in \Sigma_0$, then $A \in \Sigma_0$

If $A \leq_m B \wedge A \notin \Sigma_0$, then $B \notin \Sigma_0$

(same is true for Σ_1)

$HALT_{TM} = \{ \langle M, w \rangle \mid M$ is a TM and M halts on input $w \}$

$A_{TM} \leq_m HALT_{TM} \iff \exists f : \langle M_1, w_1 \rangle \to \langle M_2, w_2 \rangle$

s.t. M_1 accepts w_1 iff M_2 halts on input w_2

$M_2(x) :=$ simulate M_1 on ~~x~~

if M_1 accepts, then accept

$w_2 = w_1$ o.w. run for ever

$HALT_{TM} \notin \Sigma_0 \quad (\neg HALT_{TM} \notin \Sigma_1)$

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$E_{TM} = \{ \langle M \rangle \mid M \in TM \text{ and } L(M) = \emptyset \}$

$A_{TM} \leq_m E_{TM} \iff \exists f : \langle M_1, w_1 \rangle \to \langle M_2 \rangle$

$w_1 \in L(M_1)$ ie M_1 accepts w_1 iff M_2 is empty

$M_2(x) :=$ simulate M_1 on w_1

if accepts, reject x

o.w., accept x

$ALL_{TM} = \{ \langle M \rangle \mid M \in TM \text{ and } L(M) = \Sigma^* \}$

$M_2(x) :=$ simulate M_1 on w_1

if accepts, accept x

o.w. reject x

$REG_{TM} = \{ \langle M \rangle \mid M \in TM \text{ and } L(M) \in REG \}$

$\langle M_1, w_1 \rangle \to \langle M_2 \rangle$ s.t. M_1 accs w_1 iff $L(M_2) \in REG$

$M_2(x) :=$ simulate M_1 on w_1

if accepts, "be regular" accept x

o.w. "not be regular" test if $x \in 0^n1^n$, accept o.w. reject

$0^n1^n0^n$

CFL_{TM} & Σ_0

$\Sigma_{0\,TM}$ & Σ_0 "not be Σ_0" A_{TM}

→ essentially, all questions about a TM are undecidable ←

$EQ_{TM} = \{ \langle M_1, M_2 \rangle \mid L(M_1) = L(M_2) \}$

$E_{TM} \leq_m EQ_{TM}$ ~~$\langle M_1, M_2 \rangle \to \langle M_3 \rangle$~~

~~s.t. $L(M_1) = L(M_2)$ iff $L(M_3)$ is $\emptyset$~~

$E_{TM}(\langle x \rangle) = EQ_{TM}(\langle x, \emptyset \rangle)$

Linear-Bounded Automata (LBAs)

— TMs w/ a $\underbrace{\text{limited}}_{\text{not a priori}}$ tape

— defn works for arbitrary

$LBA = \{Q, \Sigma, \Gamma, q_0, \delta : Q \times \Gamma \Rightarrow Q \times \Gamma \times \{L, R\}, q_a, q_r\}$

semantics almost same — no rule for adding blanks

TM: $u[q_i]v \Rightarrow \sqcup u[q_i]v\sqcup$

$w \in L(M)$ iff $[q_0]w \Rightarrow^* u[q_a]v$

LBAs: $\boxed{\text{no rule like that}}$

$\sqcup[q_0]w\sqcup \Rightarrow^* u[q_a]v$

All examples in Σ_0, were all LBAs

$w\#w$ $\quad A_{DFA}, A_{CFG}, E_{DFA}, E_{CFG}, \quad DFA \subset LBA$

$0^n1^n0^n$

$A_{LBA} \in \Sigma_0 \quad \in LBA_0$

$\sqcup[q_0]w\sqcup \Rightarrow \sqcup a[q_1]v\sqcup \Rightarrow \sqcup[q_2]abu\sqcup$

set of LBA configurations is finite

TM config $= \Gamma^* \times Q \times \Gamma^*$

LBA config $= \Gamma^n \times Q \times \Gamma^m$ where $n + m = |w| + 2$

$A_{LBA}(\langle M, w\rangle) :=$ simulate M on w for x steps

if acc $\Rightarrow$ acc

rej $\Rightarrow$ rej

if last longer $\cancel{x}$ steps $\Rightarrow$ rej

$x = |Q| \times |w| \times |\Gamma|^{|w|+2}$

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E_{LBA} & Σ_0

$E_{LBA} = \{ \langle M \rangle \mid M \in LBA \wedge L(M) = \emptyset \}$

$\neg A_{TM} \leq_m E_{LBA}$ $\quad f : \langle M_1, w_1 \rangle \to \langle M_2 \rangle$

$M_1 \overset{\text{dne}}{\text{accepts}}\ w_1$ iff $L(M_2) = \emptyset$

$M_2(x) :=$ check that $x = \langle c_0 \rangle \langle c_1 \rangle \langle c_2 \rangle \ldots \langle c_n \rangle$

s.t. $c_0 = [q_0 \text{ from } M_1]\, w_1$

$c_n = u\, [q_a \text{ from } M_1]\, v$

and $\forall i.\ c_i \Rightarrow c_{i+1}$ according TM rules

x is "evidence that M_1 accepts w_1"

x doesn't exist (ie $L(M_2) = \emptyset$)

$\Rightarrow$ "M_1 doesn't accept w_1"

$A_{TM} \leq_m$

ALL_{CFG} Also can represent TM histories