9-2

$\forall A \in TM_1 \,.\, \exists B \in TM_2 \,.\, L(A) = L(B)$ (trivial)

$\forall B \in TM_2 \,.\, \exists A \in TM_1 \,.\, L(A) = L(B)$

Need a "bookmark" to note where in "other" tape you are

↳ bookmark is REALLY part of Γ

$\Gamma' = \Gamma \times \{\text{bookmark!}, \text{not}\}$

(0, book!) (1, not)

$\cup \{\text{cover}\}$

bookmarked?

$\Gamma' = \Gamma \cup \hat{\Gamma}$ $\quad 0\ \hat{0}\ 1\ \hat{1}\ \sqcup\ \hat{\sqcup}$

$\cup \{\#\}$ $\quad \#$

infinite ␣s

2-config

+ infinite ␣s

abcd $\left[q_i\right]$ efghi → $[q_i]$ abcd$\hat{e}$fghi # 012$\hat{3}$4567 #

012 34567

$[q_i/e]$

$\delta(q_i, (e,3)) \Rightarrow (q_j, (x,R), (4,L))$ $\quad [q_i/e/3]$

1 $\quad n + O(1)$

abcdx $\left[q_j\right]$ fghi → $[q_j]$ abcdx$\hat{f}$ghi # 01$\hat{2}$44567 #

01 244567

Steps: move right to read left tape

move right to read right tape] copy everything over and put in ␣s to simulate infinite ␣s

on right, make change & move bookmark

move left back to left bookmark

on left, make change & move bookmark

go all way left

float → (s, m, e)

$(s_x, m_x, e_x) \cdot (s_y, m_y, e_y)$

$= (s_x \cdot s_y,\ m_x \cdot m_y,\ e_x + e_y)$

$(s_x, m, e) = s \times m \times 2^e$

$[-1, +1]$

abc $\left[q_i\right]$ def → $[q_i]$ abc$\hat{\sqcup}$ # def #

abc␣ $\left[q_j\right]$ ␣def $\quad$ $[q_j]$ abc␣$\hat{\sqcup}$ # $\hat{\sqcup}$def #

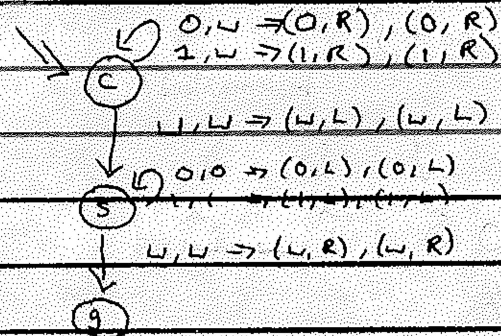
Goal: Show Σ_0 / Σ_1 closed under $\cup / \cap$

imagine TMs w/ two tapes

$$\delta : Q \times (\underset{\text{left}}{\Gamma} \times \underset{\text{right}}{\Gamma}) \longrightarrow Q \times \left(\underset{\text{left}}{(\Gamma \times \{L,R\})} \times \underset{\text{right}}{(\Gamma \times \{L,R\})} \right)$$

$u \,[q]\, v$ ← tape 1 (left)
$u' \,[\;]\, v'$ ← tape 2 (right)

copy from left to right



$[c]\, w$

$\begin{matrix} w \\ w \end{matrix} [s]$

$[g] \begin{matrix} w \\ w \end{matrix}$

combine TM_x and TM_y into TM_z s.t.

$$L(TM_z) = L(TM_x) \cup L(TM_y)$$

$Q_z = \{c, s, a, r\} \cup \cancel{Q_x Q_y}\ \delta_z$ $\qquad \Sigma_z = \Sigma_x = \Sigma_y$

$q_{0z} = c \qquad g = (q_{0x}, q_{0y})$ $\qquad \Gamma_z = \Gamma_x \cup \Gamma_y$

$\delta_z((q_i x, q_i y), (a_x, a_y)) =$ $\qquad \delta_x(q_i x, a_x) = (q_j x, b_x, d_x)$

$(q_j x, q_j y), (b_x, d_x), (b_y, d_y)$ $\qquad \delta_y(q_i y, a_y) = (q_j y, b_y, d_y)$

$\cup \quad \delta_z((q_{ax}, *), (*, *)) = (q_a, ??, ??)$

$\cap \quad \delta_z((q_{ax}, q_{ay}), (*, *)) = (q_a, ??, ??)$

both $\quad \delta_z((q_{rx}, q_{ry}), (*, *)) = (q_r, ??, ??)$

$\Sigma_0 \cup \Sigma_0 \in \Sigma_0$ (same for $\cap$)

$\Sigma_1 \cup \Sigma_1 \in \Sigma_1$

$\Sigma_0 \cup \Sigma_1 \in \Sigma_1$ $\qquad \Sigma_0 \cap \Sigma_1 \in \Sigma_0$