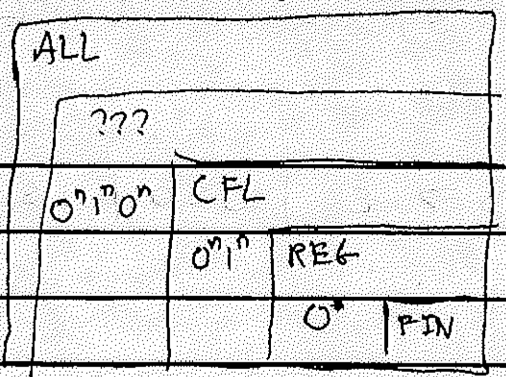
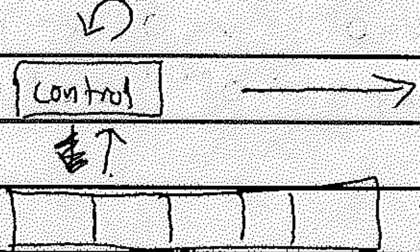
16-4



FIN terminals

∪, ∘

DFAs = NFAs



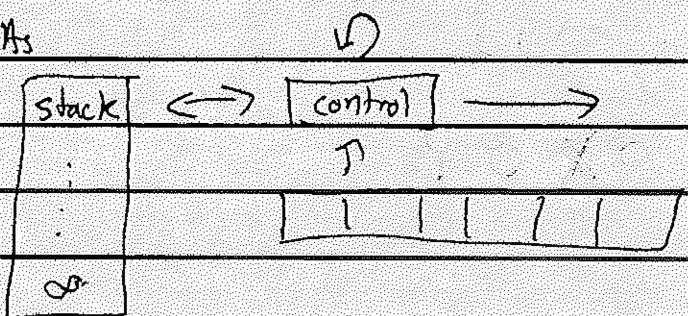
REX terminals

∪

∘

* (restricted recursion self-recursion)

PDAs



CFGs terminals, ∪, ∘

arbitrary reference

(mutual recursion)

2-stacks

different data structures

unrestricted recursion?

NFAs = 0-PDA

PDA = 1-PDA

= 2-PDA

= 3-PDA

= 4-PDA

∞-PDA

16-3

$C = \{a^i b^j c^k \mid 0 \le i \le j \le k\} \notin CFL$

palindrome $\in$ CFL $(S \to \varepsilon \mid 0S0 \mid 1S1 \mid 0 \mid 1$

$(w \mid w = w^R)$

$\{ww \mid w \in \Sigma^*\} \notin CFL$

$0^p 1 0^p 1$ $\quad u = 0^{p-1}$ $\quad v = 0$ $\quad x = 1$ $\quad y = 0$ $\quad z = 0^{p-1}1$

$uv^i x y^i z = 0^{p+i-1} 1 0^{p+i-1} 1$

$0^p 1^p 0^p 1^p$

$|x| = |y|$

$\{xy \mid x, y \in \Sigma^*, x \neq y\}$

$S \to 0 \cancel{X} 0 Y$

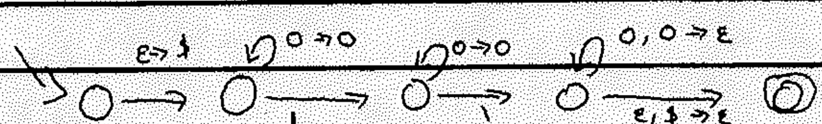
$S \to 0S1S$
$\mid 1S0S$
$\mid \varepsilon$

$\{w\#t \mid t = xwy, \ x, y, w, t \in \Sigma^*\} \notin CFL$

$0^n 1 0^m 1 0^{n+m}$ (unary addition)

$w = 0^p 1 0^p 1 0^{2p}$ $\quad u = 0^p 1 0^{p-1}$ $\quad v = 0$ $\quad \cancel{x} = 1$ $\quad y = 0$

$z = 0^{2p-1}$



$S \to 0S0 \mid 1X$

$X \to 0X0 \mid 1$

6-2

case 3 : $vxy = a^m b^p c^n$

$u = a^l \quad vxy = a^m b^p c^n \quad z = c^q$

$|vy| > 0 \qquad |vxy| < p \qquad m + p + n < p$

$\Rightarrow m = 0 = n$

impossible

case 2(3) : vxy contains only a&c

$vxy = a^m c^n \qquad p = 0 \qquad w = \varepsilon \qquad |vxy| \neq 0 \quad > 0 \rightarrow$ impossible

case 2(1) : vxy contains only a&b

$u = a^l \quad vxy = a^m b^n \quad z = b^q c^p$

$l + m = p \qquad n + q = p \qquad m + n < p$

case 2(1)(1) : v contains only a

case 2(1)(2) : v contains a & b

$v = a^{m_1} \quad x = a^{m_2} b^{n_1} \quad y = b^{n_2}$

$m_1 + m_2 = m \qquad n_1 + n_2 = n \qquad uv^i xy^i z =$

$a^{(l + m_1 i + m_2)} b^{(n_1 + n_2 i + q)} c^p \in A$

Coq

iff $l + m_1 i + m_2 = n_1 + n_2 i + q = p$

iff $i = 1$

$a^n b^n c^n \notin CFL \qquad ALL \neq CFL$

$a^n b^n c^n$ abbcabc $\notin CFL$

$(B \in REG, B \neq \emptyset) \Rightarrow \{a^n b^n c^n\} \circ B \notin CFL$

$w \in A \circ B$ iff $w = w_1 w_2 \wedge w_1 \in A \wedge w_2 \in B$

16-1

$CFPP(A) :=$ $\forall A \in CFL.\ CFPP(A)$

$\exists p \in N.$

$\forall (w \in A \mid |w| > p).$ $S \to uRz$

$\exists (u,v,x,y,z \in \Sigma^* \mid$ $R \to vRy$

$w = uvxyz$ $R \to x$

$\wedge\ |vy| > 0$ g'

$\wedge\ |vxy| < p)$ $L(g') \subseteq A$

$\left(\forall i \in N.\ uv^i x y^i z \in A\right)$

$\neg CFPP(A) :=$ $\forall$ = given $\exists$ = choose

$\forall p \in N.$ — accept any p

$\exists (w \in A \mid |w| > p)$ — choose a w ("attack")

$\forall (u,v,x,y,z \in \Sigma^* \mid$ — accept any division of w

$w = uvxyz$

$\wedge\ |vy| > 0$

$\wedge\ |vxy| < p)$

$\exists (i \in N).$ — choose a violating i

$uv^i x y^i z \notin A$

$\Sigma = \{a,b,c\}$ $aabbcc \in B$ $abbcc \notin B$

$B = a^n b^n c^n$ for $n \in N$

given: p given: u, v, x, y, z

choose: $w = a^p b^p c^p$ case 1: vxy contain $a, b,$ xor c

case 2: vxy contain $a \& b,\ b \& c,\ a \& c$

case 1a: $vxy = a^m$ case 3: vxy contain $a, b, \& c$

$u = a^l$ $vxy = a^m$ $z = a^n b^p c^p$

$l + m + n = p$ $m < p$ $m_1 + m_2 + m_3 = m$ case 1.b

$v = a^{m_1}$ $x = a^{m_2}$ $y = a^{m_3}$ $m_1 + m_3 > 0$ case 1.c

$uv^i x y^i z = a^l a^{m_1 \times i} a^{m_2} a^{m_3 \times i} a^n b^p c^p$

$= a^{l + m_2 + n + (m_1 + m_3) \times i} b^p c^p$

$l + m_2 + n + (m_1 + m_3) i = p = l + m_2 + n + (m_1 + m_3) 1$

$i = 1$