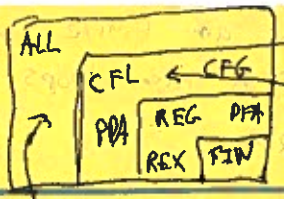
5-1



CFL ≠ ALL

$x = 0^n 1^n \in$ ALL but $\notin$ REG

$y \in$ ALL but $y \notin$ CFL

1. $\forall A \in REG.\ P(A)$ — $\forall H \in human.\ has\text{-}soul(H)$

2. $\neg P(0^n 1^n)$ — $\neg\ has\text{-}soul(toast)$

$0^n 1^n \notin REG$ $\Rightarrow$ toast $\in$ human

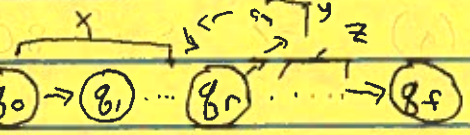
FPP — RPP(A) = regular pumping property — "DFAs have loops"

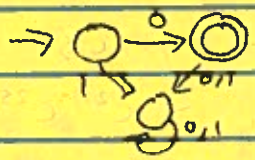
$\exists p \in N.$ — $\exists p \in N.$

$\forall (w \in A \mid |w| > p).$ — $\forall (w \in A \mid |w| > p)$

$\exists (x,y,z \in \Sigma^* \mid w = xyz \wedge |y| > 0 \wedge |xy| < p)$

$\forall (u,v,x,y,z \in \Sigma^* \mid w = uvxyz \wedge |vy| > 0 \wedge |vxy| < p).$ — $\forall i \in N.$ $xy^i z \in A$





$\forall i \in N.$ $uv^i xy^i z \in A$

$p = 3$ for all $w \in \{\}$, $|w| > 3$

$= \{\}$

Proof System = Theory — Truth system = model

soundness: Theory → Model — 1931/1921 Kurt Gödel

completeness: Model → Theory

Law of Excluded Middle: $\forall p.\ p \vee \neg p$ (correct for Truth not Proof)

$\forall p.\ \neg\neg p \rightarrow p$

$\neg(P \vee Q) \iff \neg P \wedge \neg Q$

Constructivism (logic w/ LoEM) — Coq

$(\forall A \in \emptyset.\ P(A))$ — $\neg(\forall A \in X.\ P(A)) \Rightarrow \exists A \in X.\ \neg P(A)$

$(\forall A \in \emptyset.\ \neg P(A))$ — $\neg(\exists A \in X.\ P(A)) \Rightarrow \forall A \in X.\ \neg P(A)$

15-2 Context-free pumping property

DFAs are finite state
⇒ DFAs have loops
⇒ RPPP

1. $\forall A \in CFL.\ CFPP(A)$
2. $\exists B \in ALL.\ \neg CFPP(B)$

$\Rightarrow B \notin CFL$

$\Sigma = \{a, b, c\}$ $\quad B = a^n b^n c^n$

$S \to \varepsilon \mid aXbYc$

$X \to \varepsilon \mid aXb$

$Y \to \varepsilon \mid bYc$

1. Assume the CFG (g) is in CNF

$\forall r \in R,\ r_1 = S \to \varepsilon$

or $r_2 = A \to a \quad A \in V,\ a \in \Sigma$

or $r_3 = A \to BC \quad B \neq S,\ C \neq S,\ B, C \in V$

$\varepsilon \in L(g) \Rightarrow S \xrightarrow{r_1} \varepsilon$

$a \in L(g) \Rightarrow S \xrightarrow{r_2} a$

$ab \in L(g) \Rightarrow S \xrightarrow{r_3} BC \xrightarrow{r_2} aC \xrightarrow{r_2} ab$

$\xrightarrow{r_3} CD \xrightarrow{r_2} xD \xrightarrow{r_2} xy$

$abc \in L(g) \Rightarrow S \xrightarrow{r_3} BC \xrightarrow{r_2} aC \xrightarrow{r_3} aDE \xrightarrow{r_2} abE \xrightarrow{r_2} abc$

$\xrightarrow{r_3} DEC \xrightarrow{r_2} aEC \xrightarrow{r_2} abC \xrightarrow{r_2} abc$

$S \to \varepsilon$

$S \to a \ldots S \to b \ldots S \to c \quad \Sigma = \{a, b, c\}$

$S \to AA \to BB\ BB \to C^4 C^4 \to D^8 D^8 \to \ldots \to C^{256} C^{256}$

$u, z \in \Sigma^*$

$v, y \in \Sigma^*$

$x \in \Sigma^*$

$S \to uRz \qquad uvxyz \in A \Rightarrow uv^i xy^i z \in A \quad \forall i \in N$

$R \to vRy$

$R \to x$

$S \to \varepsilon \mid 0S1 \qquad 00001111 \in 0^n 1^n$

$E \to 0 \mid 1 \mid E+E \mid E \times E$

$1 + 1 \times 1 + 1$

$S \to 000R111 \quad u = 000$

$R \to 0R1 \quad v = 0$

$R \to \varepsilon \quad x = \varepsilon$

$y = 1$

$z = 111$

$S \to \varepsilon R + 1$

$R \to 1 + R\ \varepsilon$

$R \to 1 \times 1$