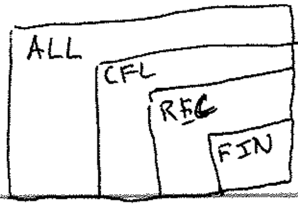
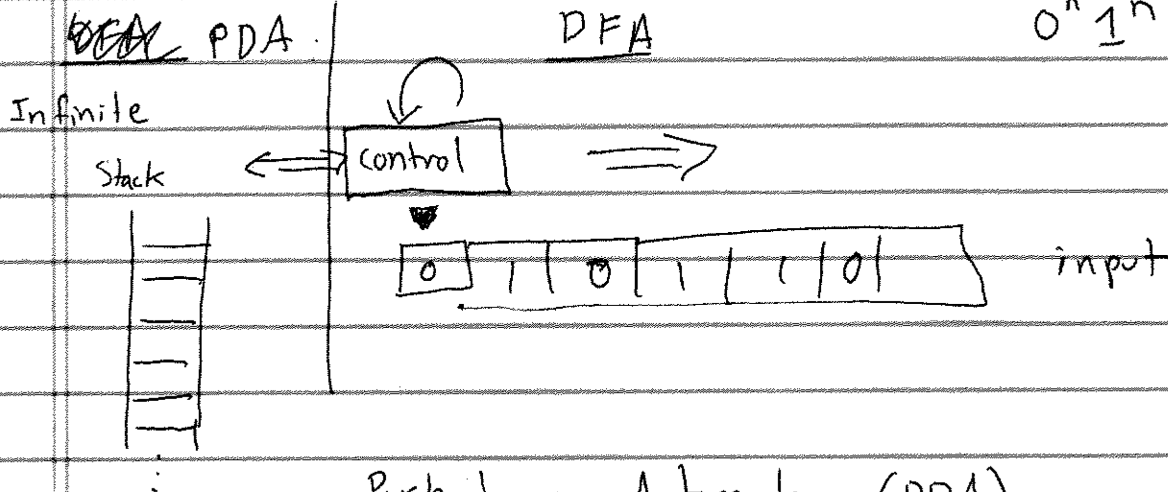
11-1



REG	DFA	REX	(NFA)
CFL	PDA	CFG	(CNF)

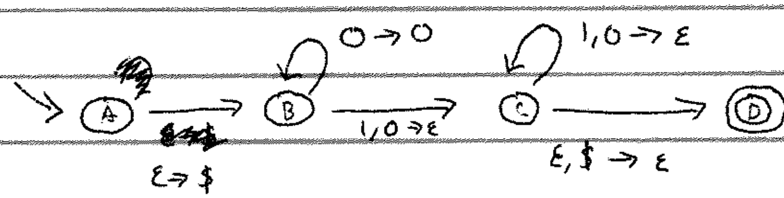


$0^n 1^n \in$ CFL

$\notin$ REG

Push-down Automata (PDA)

PDA for $0^n 1^n$



finite state

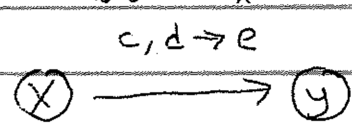
unique start

many accepting

Γ is stack alphabet

$\Gamma = \{0, \$\}$

transition from x to y



$c \in \Sigma \cup \{\varepsilon\}$ $\qquad c \to e \quad (d = \varepsilon)$

$e, d \in \Gamma \cup \{\varepsilon\}$ $\qquad c \quad (e = d = \varepsilon)$

ignore $\quad c, \varepsilon \to \varepsilon$

push $\quad c, \varepsilon \to e$

pop $\quad c, d \to \varepsilon$

replace $\quad c, d \to e$

like NFAs, use ε and be non-det

PDA config = $\underbrace{\Gamma^*}_{\text{stack}} \times \underbrace{Q}_{\text{current state}} \times \underbrace{\Sigma^*}_{\text{input}}$

$g[B]w$

$\varepsilon[A]0011 \Rightarrow \$[B]0011 \Rightarrow \$0[B]011 \Rightarrow \$00[B]11 \Rightarrow$

$\$0[C]1 \Rightarrow \$[C] \Rightarrow [D]$ ✓

$\varepsilon[A]011 \Rightarrow \$[B]011 \Rightarrow \$0[B]11 \Rightarrow \$[C]1 \Rightarrow [D]1$ X

$\varepsilon[A]001 \Rightarrow \$[B]001 \Rightarrow \$0[B]01 \Rightarrow \$00[B]1 \Rightarrow \$0[C]$ X

11-2/ PDA semantics

$P = (Q, \Sigma, \Gamma, q_0, \delta, F)$

Q is a finite set of states

Σ is an alphabet for input

Γ is an alphabet for stack

$\delta : Q \times \Sigma_\varepsilon \times \Gamma_\varepsilon \to P(Q \times \Gamma_\varepsilon)$ $\quad q_0 \in Q \quad F \subseteq Q$

current state, input, pre-stack, next state, post stack

$\Sigma_\varepsilon = \Sigma \cup \{\varepsilon\} \qquad \Gamma_\varepsilon = \Gamma \cup \{\varepsilon\}$

$w \in \Sigma^*$ $\qquad g \in \Gamma^*$

$w \in L(p)$ iff $\varepsilon[q_0]w \Rightarrow^* g[q_f]\varepsilon$ where $q_f \in F$

$$\frac{}{g[q]\varepsilon \Rightarrow^* g[q]\varepsilon} \qquad \frac{(q_j, e) \in \delta(q_i, c, d)}{gd[q_i]cw \Rightarrow^* ge[q_j]w}$$

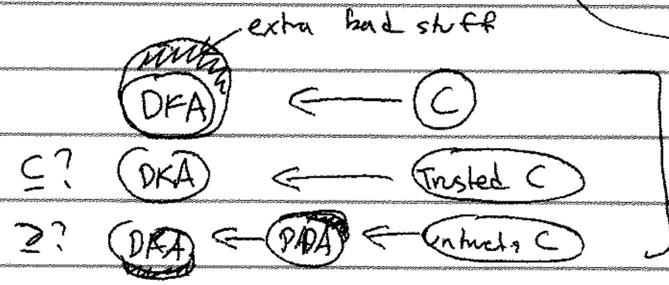
$g \in \Gamma^*$ $\quad w \in \Sigma^*$ $\quad q_i, q_j \in Q$ $\quad c \in \Sigma_\varepsilon$ $\quad d, e \in \Gamma_\varepsilon$

PDAs don't exist

but can specify

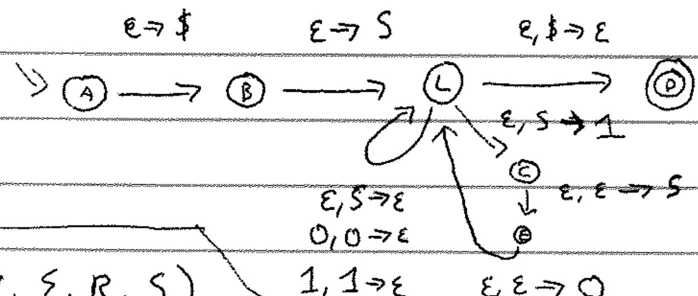
families of DFA

by restricting stack



model checking

$\forall g \in CFG.\ \exists p \in PDA.\ L(g) = L(p)$ $\qquad S \to \varepsilon \mid 0S1$



$\varepsilon[A]0011 \Rightarrow \$[B]0011$
$\Rightarrow \$S[L]0011 \Rightarrow \$1[C]0011$
$\Rightarrow \$1S[E]0011 \Rightarrow \$1S0[L]0011$
$\Rightarrow \$1S[L]011 \Rightarrow \$11[C]011$
$\Rightarrow \$11S[E]011 \Rightarrow \$11S0[L]011$
$\Rightarrow \$11S[L]11 \Rightarrow \$11[L]11$
$\Rightarrow \$1[L]1 \Rightarrow \$[L] \Rightarrow [D]$ ✓

$g = (V, \Sigma, R, S)$

$P = (Q, \Sigma, \Gamma, q_0, \delta, F)$

$\Gamma = \{\$\} \cup \Sigma \cup V \qquad \Sigma = \Sigma$

$F = \{q_f\}$

$\delta(q_0, \varepsilon, \varepsilon) = \{(q_1, \$)\}$

$\delta(q_1, \varepsilon, \varepsilon) = \{(q_L, S)\}$

$\forall c \in \Sigma.\ \delta(q_L, c, c) = \{(q_L, \varepsilon)\}$

$\delta(q_L, \varepsilon, \$) = \{(q_f, \varepsilon)\}$

$Q = \{q_0, q_1, q_L, q_f\} \cup V\ (q_v)$

if $S \to \varepsilon \in R$ then $\delta(q_L, \varepsilon, S) \ni (q_L, \varepsilon)$

if $A \to a \in R$ then $\delta(q_L, \varepsilon, A) \ni (q_L, a)$

if $A \to BC \in R$ then $\delta(q_L, \varepsilon, A) \ni (q_B, C)$

$\forall v \in V.\ \delta(q_v, \varepsilon, \varepsilon) \ni (q_L, v)$