)-4 / Regular ... DFA → NFAs → nice to program
→ make proofs easy

CFL ... CFG → CNF
Chomsky Normal Form
is like assembly for CFG

CNF is CFG but w/ restrictions

Every rule is either ...

| | | |
|---|---|---|
| $A \to BC$ | $A, B, C \in V$ | and $B \neq S$ |
| $A \to a$ | $a \in \Sigma$ | $C \neq S$ |
| $S \to \varepsilon$ | | |

---

$S \to \varepsilon \mid 0S1$

remove S from RHS

$S' \to S$
$S \to \varepsilon \mid 0S1$

pushed back epsilon

$S' \to S \mid \varepsilon$
$S \to 0S1 \mid 01$

remove unit rules

$S' \to 0S1 \mid 01 \mid \varepsilon$
$S \to 0S1 \mid 01$

introduce more variables for $\varepsilon$ and $\geq 2$

$S' \to XA \mid XY \mid \varepsilon$
$S \to XA \mid XY$
$A \to SY$
$X \to 0 \quad Y \to 1$

10-3/

Reg is closed under
- union
- intersect
- concat
- star
- complement
- reversal

CFL closed under...
- union
- concat
- star
- unambiguify

If $G_0 \in CFL$ and $G_1 \in CFL$, then...

$G_0 \cup G_1 \in CFL$

$G_2$: $S \to G_0.S \mid G_1.S$

$G_2 = (V_2, \Sigma, R_2, S_2)$

$V_2 = (V_0 \cup V_1 \cup \{S_2\}$

$R_2 = (R_0 \cup R_1 \cup \{S_2 \to S$

$\quad \mid S_2 \to S$

$G_0 \circ G_1 \in CFL$

$S \Rightarrow G_0.S \circ G_1.S$

$G_0^* \in CFL$

$S \to \varepsilon \mid G_0.S \circ S$

---

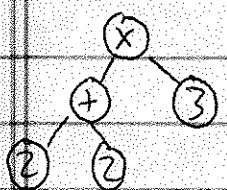
Ambiguity - differents path lead to same place

$E = \overset{A}{0} \mid \overset{B}{1} \mid \overset{C}{E+E} \mid \overset{D}{E \times E} \mid \overset{F}{2} \mid \overset{F}{3}$ $\quad 2 = 1+1$

$2 + 2 \times 3$ $\quad 3 = 2+1$

CFDFG

DCFFG




E

C ↙ ↘ D

E+E $\quad$ E×E

F ↙ $\quad$ ↘

2+E $\quad$ E+E×E

↙D $\quad$ ↙F

2+E×E $\quad$ 2+E×E

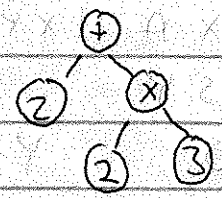
↙F $\quad$ ↙F

2+2×E $\quad$ 2+2×E

↙G $\quad$ ↙

2+2×3 $\quad$ 2+2×3

CFDFG




unambiguify DOES NOT EXIST

2-2 / Semantics of CFGs

$w \in L(g)$ iff $S \Rightarrow^* w$ ($S \in V$, $w \in \Sigma^*$)

String $A$ derives string $B$ ($A \Rightarrow^* B$) $A, B \in (V \cup \Sigma)^*$

iff

$$\frac{}{A \Rightarrow^* A} \qquad \frac{A \Rightarrow B \quad B \Rightarrow^* C}{A \Rightarrow^* C}$$

String $A$ yields string $B$ ($A \Rightarrow B$) $A, B \in (V \cup \Sigma)^*$

$(\hat{V}, w) \in R$ $\hat{V} \in V$ $x \in \Sigma^*$

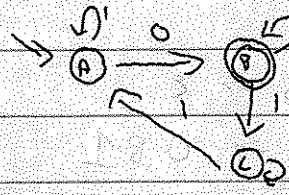
$x\hat{V}y \Rightarrow xwy$ $y, w \in (V \cup \Sigma)^*$

REG vs CFL

CFL $\subsetneq$ REG $B \in$ CFL, $B \notin$ REG

REG $\subset$ CFL?

$\forall r \in$ REG. $\exists g \in$ CFG, $L(r) = L(g)$




1001010

$V = Q$ $S = q_0$

$\forall q_f \in F$, $(q_f, \varepsilon) \in R$

$\delta(q_i, a) = q_j \Rightarrow (q_i, aq_j) \in R$

$q_0 \Rightarrow^* w q_i$ iff $[q_0]w \Rightarrow^* [q_i]$

$V = A, B, C$

$S = A$

$R = A \Rightarrow 1A \mid 0B$

$B \Rightarrow \varepsilon \mid 0B \mid 1C$

$C \Rightarrow 1A \mid 0C$

$B \Rightarrow^x \varepsilon$

$0B1 \Rightarrow^x 01$

$00B11 \Rightarrow 00B\#1$

$000B111 \Rightarrow 000111$

$0^4B1^4 \Rightarrow 0^41^4$

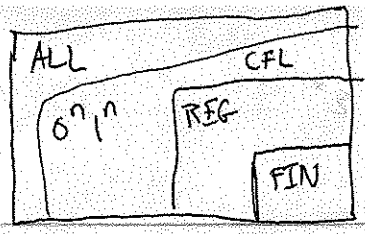
$0^5B1^5 \Rightarrow 0^51^5$

$A \Rightarrow 1A \Rightarrow 10B \Rightarrow 100B$

$\Rightarrow 1001C \Rightarrow 1001 0C \Rightarrow 10010A$

$\Rightarrow 1001010B \Rightarrow 1001010$

10-1




CFL – context-free languages
= PDA, CFG
REG = DFAs, REX

Valid English = Valid Subject ∘ Valid Verb ∘ Valid Object
| VS ∘ VV
VV = runs | jumps | hops | talks
VS = Jay | He | Her | cat
| Adj ∘ VS
Adj = Pink | Brown-haired | ...

Valid Math = Formula ∘ = ∘ Formula
Formula = Formula ∘ Operation ∘ Formula
| Number
Operation = + | − | × | ÷
$1+1 = 17 \in$ Valid Math

---

B = ε | 0 B 1

start symbol; rule RHS; terminal; symbol

B
x B → ε
y B → 0 B 1

$0^3 1^3 \in B$
$000111 \in B$

$B \xrightarrow{y} 0B1 \xrightarrow{y} 00B11 \xrightarrow{y} 000B111$
$\xrightarrow{x} 000\varepsilon 111 = 000111 = 0^3 1^3$

CFG – context-free grammar — Noam Chomsky

$g = (V, \Sigma, R, S)$ $\quad V \cap \Sigma = \emptyset$

V = a finite set of symbols/variables $\quad \Sigma$ = alphabet

$S$ = start symbol $\in V$ $\quad R$ = rules

$\in P(V \times (V \cup \Sigma)^*)$