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Foundation of Com Sci

Theory of Computation

Hierarchy of Sets — Hier of Questions

Simple sets
⇒ Basic sets
⇒ Complicated sets
⇒ Exotic sets
⇒ Weird sets

"run"

The definition of sets
- What is a set?

$\{a, b, c\}$
$\{b, a, c\}$

a	✓
b	✓
c	✓
d	X
e	X
f	X
⋮	⋮

U — the universe

{☺, ☺, ☺}

Finite sets — can be written as list of elements

$\in$ - set membership

$\emptyset$ — empty set $\quad \forall x.\ x \notin \emptyset$

$a \in \{a, b, c\}$ ✓

$a \in \emptyset$ X

$\cup$ — union operation : Set × Set ⇒ Set

$\forall x.\ x \in A \cup B$ iff $x \in A$ or $x \in B$

$\cap$ - intersection : $\forall x.\ x \in A \cap B$ iff $x \in A$ and $x \in B$

$\subset$ - subset : Set × Set → Prop. $A \subset B$ iff $\forall x.\ x \in A \Rightarrow x \in B$

P - power set $(P(A), \mathbb{P}(A), 2^A)$ $\quad P(\{☺, ☺\}) =$

$\forall x.\ x \in P(A)$ iff $x \subset A$ $\quad \{\{\}, \{☺\}, \{☺\}, \{☺, ☺\}\}$

$P(\emptyset) = \{\emptyset\}$ $\quad P(P(\emptyset)) = \{\emptyset, \{\emptyset\}\}$ $\quad P^n(\emptyset)$ = a set with n things

relations and functions

$f : X \Rightarrow Y$ $\quad f \subset \underbrace{X \times Y}_{\text{cartesian product}}$ $\quad x \in A \times B$ iff $\underbrace{x = (a, b)}_{\text{pair}}$

dom ↗ range ↗

$\boxed{f(x) = y \Leftrightarrow (x, y) \in f}$ and $a \in A$

$r : X \times Y$ — $r \subset X \times Y$ $\quad \forall x, y_1, y_2$ and $b \in B$

$f(x) = y_1 \wedge f(x) = y_2$

$\Rightarrow y_1 = y_2$

1-2 Pairs like $(a,b) \in A \times B$ are a special case of tuples, 2-tuples

$(A \times B) \times C \ni ((a,b),c)$

String — "a list of things"

list ↳ ? either null or a thing and a list

a function from N (natural numbers) to things (and a N)

abc

$:= (3, \; 0 \to a \;\; 1 \to b \;\; 2 \to c)$

A string is a list of characters (elements of set Σ) sigma

= alphabet

ϵ — lower case Greek epsilon = the string of length 0

x_i — the ith character in x

$|x|$ — the length of string x

$x \circ y$, xy — the concatenation

$(xy)_i = x_i$ if $i < |x|$

y_j where $j = i - |x|$

lexicographic ordering of strings, $\Sigma = \{0,1\}$

$\epsilon, 0, 1, 00, 01, 10, 11, 000, 001, 010, 011, \ldots$

language of an alphabet is a set of strings

$\{0, 01\}$

"All binary strings that represent odd numbers"

$\{0, 01\} \cup \{1, 11\} = \{0, 1, 01, 11\}$

$X \cup Y \cap Z \times A =$ "odd numbers"?