

-1/

$M := X \mid b \mid \lambda X.M \mid (o^n M \dots)$       $V := b \mid \lambda X.M$       $B_V : (\lambda X.M)V \rightarrow M[X \leftarrow V]$   
 $\mid M \ N$       $\mid (o^n M \dots)$       $\Delta : (o^n b \dots) \Rightarrow \delta(o^n, b \dots)$

$E = \bullet \mid E \ N \mid V \ E \mid o^n \ V \dots \ E \ M \dots$

$K = \text{ret} \mid \text{fn}(N, k) \mid \text{ar}(V, k) \mid \text{pr}(o^n, \langle V \dots \rangle, \langle M \dots \rangle, k)$   
 "stack"     Continuation = future of computation

$\langle M \ N, k \rangle \xrightarrow{\text{ck}} \langle M, \text{fn}(N, k) \rangle$   
 $\langle V, \text{fn}(N, k) \rangle \xrightarrow{\text{ck}} \langle N, \text{ar}(V, k) \rangle$   
 $\langle V, \text{ar}(\lambda X.M, k) \rangle \xrightarrow{\text{ck}} \langle M[X \leftarrow V], k \rangle$

$P \xrightarrow{\text{ck}} \dots \xrightarrow{\text{ck}} \langle X, k \rangle$       $X$  is unbound

$\langle ((\lambda X. (\lambda Y. (\lambda Z. P))) \ 4) \ 5) \ 6), k \rangle \xrightarrow{\text{ck}^*}$   
 $\langle P[X \leftarrow 6][Y \leftarrow 5][Z \leftarrow 4], k \rangle$

$P = X$       $P = ((\lambda X. 9) \ 9) \ (\lambda Y. (X(YZ)))$

$\text{CEK} \rightarrow \text{environment } \Sigma \mid \Sigma = V \bullet \mid \Sigma[X \mapsto V]$

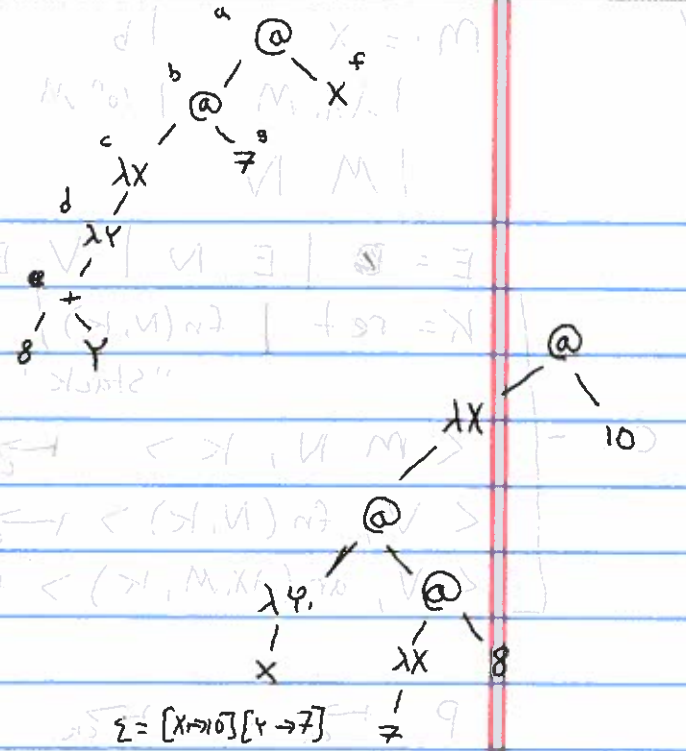
broken CEK  
 $\langle X, \Sigma, k \rangle \xrightarrow{\text{CEK}} \langle V, \Sigma, k \rangle$  where  $[X \mapsto V] \in \Sigma$   
 $\langle M \ N, \Sigma, k \rangle \xrightarrow{\text{CEK}} \langle M, \Sigma, \text{fn}(N, k) \rangle$   
 $\langle V, \Sigma, \text{fn}(N, k) \rangle \xrightarrow{\text{CEK}} \langle N, \Sigma, \text{ar}(V, k) \rangle$   
 $\langle V, \Sigma, \text{ar}(\lambda X.M, k) \rangle \xrightarrow{\text{CEK}} \langle M, \Sigma[X \mapsto V], k \rangle$

- PHP - mathjs  
 - elisp - shells

dynamic scope     the place where variables are bound  
 depends on execution

JS:  $o.m(a) \Leftrightarrow o[m](o, a)$   
 $o[m](a) \text{ --- } o[m](this, a)$

9-2)  $((\lambda x. (\lambda y. 8 + y) ) 7) x$



- < a, ~~ret~~, ret >
- < b, •, fn(f, ret) >
- < c, •, fn(g, fn(f, ret)) >
- < g, •, ar(c, fn(f, ret)) >
- < d, • [X → 7], fn(f, ret) >
- < f, • [X → 7], ar(d, ret) >
- < 7, • [X → 7], ar(d, ret) >
- < e, • [X → 7] [Y → 7], ret >
- = < 8 + Y, ret >
- < Y, • [X → 7] [Y → 7], pr(+, < 8 >, < >, ret) >
- < 7, " " >
- < 15, " " , ret >

CEK

$[k = \text{ret} \mid \text{fn}(N, \Sigma, k) \mid \text{ar}(V, k)]$

- $\exists \langle X, \Sigma, k \rangle \xrightarrow{\Sigma_{EK}} \langle V, \Sigma, k \rangle$  where  $[X \mapsto V] \in \Sigma$
- $\langle M N, \Sigma, k \rangle \xrightarrow{\Sigma_{EK}} \langle M, \Sigma, \text{fn}(N, \Sigma, k) \rangle$
- $\langle V, \Sigma, \text{fn}(N, \Sigma', k) \rangle \xrightarrow{\text{CEK}} \langle N, \Sigma', \text{ar}(V, k) \rangle$
- ~~$\langle V, \Sigma, \text{ar}(\lambda X.M, k) \rangle \xrightarrow{\text{CEK}} \langle M, \bullet [X \mapsto V], k \rangle$~~

static  
scope

$V = b \mid \text{clo}(\lambda X.M, \Sigma)$

$\langle \lambda X.M, \Sigma, k \rangle \xrightarrow{\text{CEK}} \langle \text{clo}(\lambda X.M, \Sigma), \Sigma, k \rangle$

$\langle V, \Sigma, \text{ar}(\text{clo}(\lambda X.M, \Sigma'), k) \rangle$

$\xrightarrow{\text{CEK}} \langle M, \Sigma'[X \mapsto V], k \rangle$

9-3)

# Implementing $\Sigma$

$$\langle X, \Sigma, K \rangle \mapsto \langle V, \Sigma, K \rangle$$

$$[X \mapsto V] \in \Sigma$$

$$\text{closure} = \text{clo}(\lambda X, M, \Sigma)$$

code pointer

variable mapping

flat closure

X

}

$$\Sigma = [X]$$

$$\text{movg } \%rax \leftarrow \%rax[0]$$

flat closure =  $\Sigma$  is an array of the mentioned variable

$$\lambda X, \dots X \dots Y \dots Z \dots \quad \Sigma = [X, Y, Z]$$

$$\downarrow$$
  
$$\text{rax}[0]$$

$$\downarrow$$
  
$$\text{rax}[4]$$

$$\downarrow$$
  
$$\text{rax}[8]$$

$$[Y, X, Z]$$

nested closure

$\Sigma$  = pointer to the last fun call

$$\Sigma = [X, Y, \rightarrow] \rightarrow [Z, A, B, \rightarrow] \rightarrow [C, \rightarrow]$$

$$X$$
  
$$\downarrow$$
  
$$\text{rax}[0]$$

$$Y$$
  
$$\downarrow$$
  
$$\text{rax}[4]$$

$$Z$$
  
$$\downarrow$$
  
$$\text{rax}[8][0]$$

X,



Σ prisms?

$$\langle X, \Sigma, K \rangle \mapsto \langle V, \Sigma, K \rangle$$

$$\Sigma \in [X \rightarrow V]$$

$$\text{prisms} = \text{clo}(\Sigma, M, X)$$

Σ prisms, M, X, V

$$[X] = \Sigma$$

Σ is an union of the prisms

$$[\Sigma, X, Y] = \Sigma$$

Σ = union of the prisms

$$[\Sigma, X, Y] = \Sigma$$