1-1/ $C = \{ w \mid w$ has equal number of 0s & 1s $\}$

Given language A, nat p

Choose $s \in A$ s.t. $|s| \geq p$

Try #1 $s = (01)^p$

Given xyz s.t. $s = xyz$, $|xy| \leq p$, $|y| > 0$

Choose $i \in N$ s.t. $xy^iz \notin A$

$xyz = (01)^p$ $x = (01)^a$ $y = (01)^b$ $z = (01)^c$ $a+b+c = p$

$a+b < p/2$ $b > 0$

① x is odd, y even, z odd

② x is odd, y odd, z even

③ x is even, y odd, z odd

④ x even, y eve, z ev

④ $x = (01)^a$ $y = (01)^b$ $z = (01)^c$

$a+b+c = p$ $b > 0$ $a+b < p/2 \rightsquigarrow 2a+2b < p$

$xy^iz = (01)^a (01)^{bi} (01)^c \in A$ iff true

FAIL our goal

Try #2

$s = 0^p 1^p$

$xyz = 0^p 1^p$ $xy = 0^a$ $z = 0^b 1^p$ $a+b = p$ $a < p$ $b > 0$

$i = 0$ $x = 0^l$ $y = 0^j$ $l+j = a$ $j > 0$

$xy^0z = 0^l 0^b 1^p \in A$ iff $l+b = p = a+b$ FALSE

$l = a = l = l+j$

$j = 0$

9-2/ $F = \{ww \mid w \in \{0,1\}^*\}$

Given : $p \in N$

Choose : $s \in F$ s.t. $|s| \geq p$

$s = 0^p 1 0^p 1$ $\quad w = 0^p 1$

Given : x, y, z s.t. $s = xyz$ $\quad |y| > 0$ $\quad |xy| \leq p$

$xyz = 0^p 1 0^p 1$ $\quad xy = 0^a$ $\quad z = 0^b 1 0^p 1$ $\quad a + b = p$

$x = 0^c$ $\quad y = 0^d$ $\quad c + d = a$ $\quad d > 0$

Choose: i s.t. $xy^iz \notin F \leftrightarrow$

$xy^iz = 0^c 0^{di} 0^b 1 0^p 1$ $\quad c + di + b = p = a + b$

$0, 2, 3,$ etc $\neq 1$ $\quad c + di = a = c + d$

$di = d$

$i = 1$

$D = \{1^{n^2} \mid n \geq 0\}$ $\quad \Sigma = \{1\}$

Given : $p \in N$

choose : $s \in D$ s.t. $|s| \geq p$

$s = 1^{p^2}$

Given : $xyz = s$ $\quad |y| > 0$ $\quad |xy| \leq p$

$xyz = 1^{p^2}$ $\quad x = 1^a$ $\quad y = 1^b$ $\quad z = 1^c$ $\quad a + b + c = p^2$ $\quad b > 0$ $\quad a + b \leq p$

Choose : $i \in N$ $\quad xy^iz \notin F$

$xy^iz = 1^a 1^{bi} 1^c$ $\quad$ if $a + bi + c = n^2$ (for some n)

$(a+b+c) + (i-1)b = n^2$

$p^2 + (i-1)b = n^2$ $\quad b < p$

$i=2$ $\quad p^2 + b = n^2$ $\quad i=3$ $\quad \boxed{p^2 + 2b = n^2}$ rely on diff between sq not even

$i=5$ $\quad p^2 + 4b = n^2$ $\quad \cancel{2b = n^2 - p^2 = (n+p)(n-p)}$

$2b = (n+p)(n-p)$

$(n+1)^2 - n^2 = n^2 + 1 + 2n - n^2 = 2n + 1$

1-3

$E = \{ 0^i 1^j \mid i > j \}$

Given: $p \in N$

Choose: $s \in E$ s.t. $|s| \geq p$

$s = 0^{2p} 1^p$

Given: $s = xyz$ $\quad |y| > 0 \quad |xy| \leq p$

$xyz = 0^{2p} 1^p \quad x = 0^a \quad y = 0^b \quad z = 0^c 1^p$

$a + b + c = 2p \quad a + b \leq p \quad b > 0$

Choose: $i \in N$ s.t. $xy^iz \notin E$

$xy^iz = 0^a 0^{bi} 0^c 1^p$

$a + bi + c > p$

$2p + (i-1)b > p$

$(i-1)b > -p \longrightarrow i - 1 > -p/b \Rightarrow i > -p/b + 1$

$-(i-1)b < p$ $\qquad p > 0 \quad b > 0$

$b - bi < p$

$s = 0^{p+1} 1^p \qquad xyz = 0^{p+1} 1^p \quad x = 0^a \quad y = 0^b \quad z = 0^c 1^p$

$a + b + c = p + 1$

$xy^iz \Rightarrow a + bi + c > p$

$i = 0 \Rightarrow a + c > p \qquad b \neq 0 \qquad a + c = p \qquad p \not> p$

$PLUS = \{ 0^n 1 0^m 1 0^{n+m} \mid n, m \in N \}$

" $n + m = \underline{n+m}$ "

Given: p

Choose: $s = 0^p 1 0 1 0^{p+1}$ $\qquad$ " $p + 1 = p+1$ "

$x = 0^a \quad y = 0^b \quad z = 0^c 1 0 1 0^{p+1} \qquad a + b + c = p$

$xy^iz = 0^{a+bi+c} 1 0 1 0^{p+1} \quad \in PLUS$ iff

$a + bi + c + 1 = p + 1$

$a + bi + c = p = a + b + c$

$bi = b$

$i = 1$

10-1/ Acceptors (DFAs) $\rightarrow$ Language Denot (REX)

$\Downarrow$ (str $\rightarrow$ t or f) $\Downarrow$ (rex $\rightarrow$ set(str))

Context-Free Languages (CFL) CFL like REG
denotation: Context-Free Grammar (CFG) CFG like REX
acceptor: Push-Down Automata (PDA) PDA like DFA

Example CFG:

$A \rightarrow 0A1$ ① $\leftarrow$ rule, or production or a substitution rule

$A \rightarrow \varepsilon$ ②

$\uparrow$ lhs $\in$ Variable $\uparrow$ rhs $\in (\text{Vars} \cup \Sigma)^*$

$\rightarrow$ terminals

$\rightarrow$ non-terminals

one variable is the start variable (A) variable on the lhs of the first rule

A "derivation" of a string of Grammar G:

$A \xrightarrow{①} 0A1 \xrightarrow{①} 00A11 \xrightarrow{①} 000A111 \xrightarrow{②} 000111$

"000111" ↛ 1112

① $A \rightarrow AB$ ② $A \rightarrow 1$ ③ $B \rightarrow 0$

$A \xrightarrow{①} AB \xrightarrow{②} 1B$

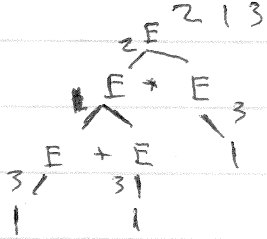
$AB \xrightarrow{③} A0$

A parse tree is the sequence of rules

1. $E \rightarrow E + E$
2. $E \rightarrow E * E$
3. $E \rightarrow 1$

What derivation produces $1 + 1 * 1$?

21333



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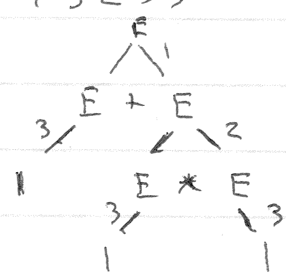

ambiguous grammar