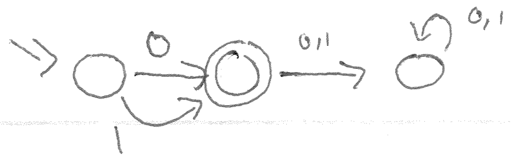
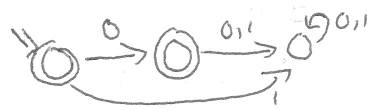
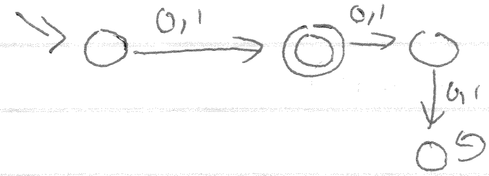
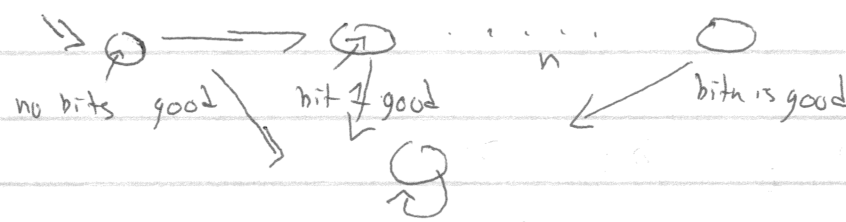
3-1 / What is the smallest DFA over Σ_{01} where $|L(d)| = 2$







What is the ~~number~~ number of states for a Language with 1 string n-bits long?

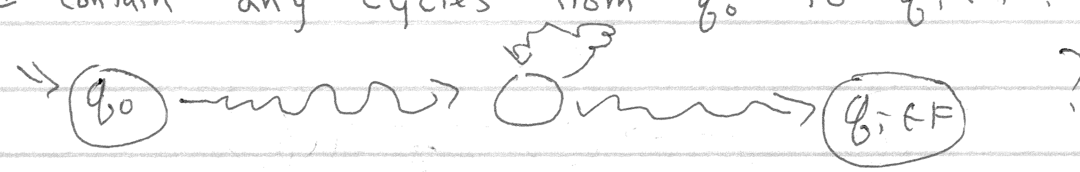


$2+n$ $(n>0)$

2 $(n=0)$

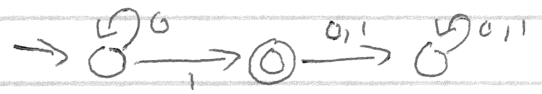
Assume language A is finite and DFA d s.t. $L(d) = A$

Does d contain any cycles from q_0 to $q_i \in F$?



↓

No, Not possible.



$L = 0^*1$) NOT Finite

Assume d has q states (eg. $q = 10$)

If $L(d)$ is finite, how long is the longest string? ≤ 8

is infinite, then anything possible, ⇒ there's a cycle

Assume A is infinite, $L(d) = A$, can d have no cycles? No.

Suppose $w \in A$ and $|w| = 10$, how many states does it visit?

≤ 11

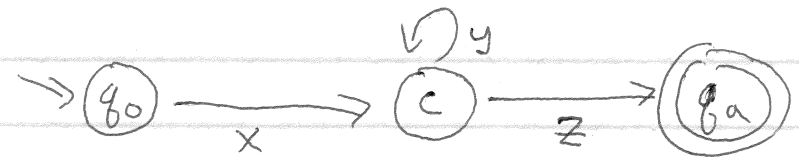
how many unique $= 11$

8-2 / F is "the regular pumping property"

$F(A) = \exists p \in \mathbb{N}.$
$\forall s \in A,$
$|s| \geq p \rightarrow$
$\exists xyz,\ s = xyz$
$\wedge\ (\forall i \geq 0,\ xy^iz \in A)$
$\wedge\ |y| > 0$
$\wedge\ |xy| \leq p$

) $\forall d \in DFA,\ F(L(d))$
) $\exists x,\ \neg F(x)$

① $d = \langle \Sigma, Q, q_0, \delta, F \rangle$
$p = Q+1$ ← we choose
given $s \in L(d)$ → visits $\leq Q+1$ states, $Q+1$ unique states
we choose x, y, z must visit 1 state twice

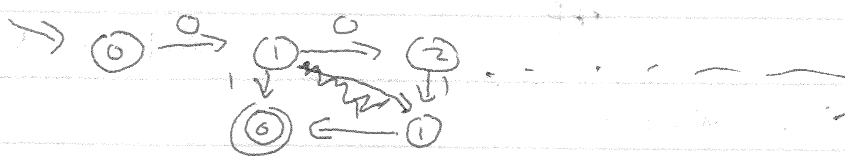


$|y| > 0$? yes atleast 1
$|xy| \leq p$? yes
$\forall i \geq 0,\ xy^iz \in A$? yes xz xyyyz boom

8-3

$B = \{0^n 1^n \mid n \geq 0\}$ $\qquad$ $B \notin REG$

suppose $n \leq 3$ – ε, 01, 0011, 000111



$\neg \exists P = \forall \neg P$

$\neg \forall P = \exists \neg P$

$\neg (P \Rightarrow Q) = P \Rightarrow \neg Q$

(2) $\neg F(B)$ RPP

$\forall p \in N,$

$\exists s \in A,$

$|s| \geq p \Rightarrow$

$\forall xyz, \ s = xyz$

$(\exists i \geq 0, \ xy^iz \notin A)$

$\vee \ |y| = 0$

$\vee \ |xy| > p$

Given $p \in N$, $xyz \in \Sigma^* \ni |y| > 0, \ni |xy| \leq p$

Give $s \in A \ni |s| \geq p$, $i \in N, \ni xy^iz \notin A$

$s = 0^{p+1} 1^{p+1}$ $\qquad$ $|s| = 2(p+1)$

$= xyz$ $\qquad$ $|y| > 0$ $\qquad$ $|xy| \leq p$

$xy = 0^a$ $\qquad$ $z = 0^b 1^{p+1}$ $\qquad$ $a + b = p + 1$

$x = 0^u$ $\quad$ $y = 0^v$ $\qquad$ $u + v = a$ $\qquad$ $v > 0$

$u + v + b = p + 1$ $\qquad$ $v > 0$

$xy^iz = 0^u 0^{iv} 0^b 1^{p+1}$ $\in A$ iff $u + iv + b = p + 1$

$(u + iv + b) - (u + v + b) = (p+1) - (p+1)$

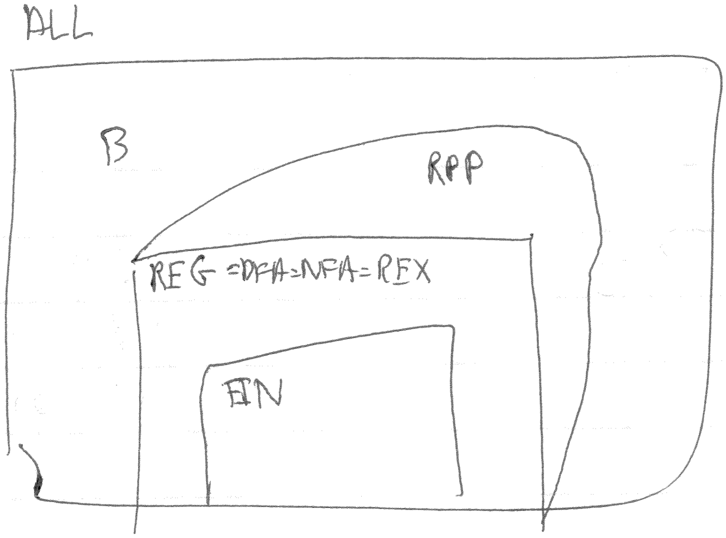
$(i-1)v = 0$

$i - 1 = 0 \Rightarrow i = 1$

choose any i except 1

∎

3-4



REG ⊂ RPP
RPP ⊄ REG

```
int c = 0;    bool onlyones = 0;
while (char n = getc(); } {
    if (onlyones) {
        if (n == '0') {
            c++;
        } else {
            c--;
            onlyones = 1;
        }
    } else }
        if (n == '0") return false;
        c--;
    }
}
return (c == 0; )
```