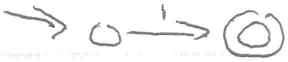
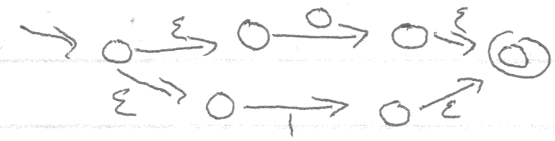
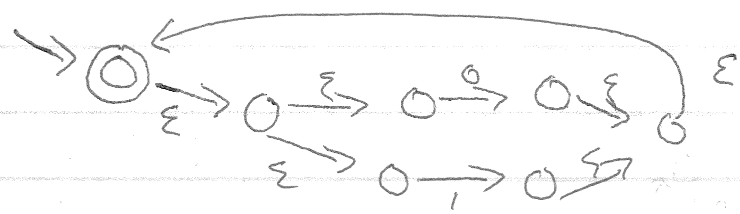
5-3/ $C(\Sigma^* 1 \Sigma \Sigma) = C(\Sigma^* \circ (1 \circ (\Sigma \circ \Sigma))) =$

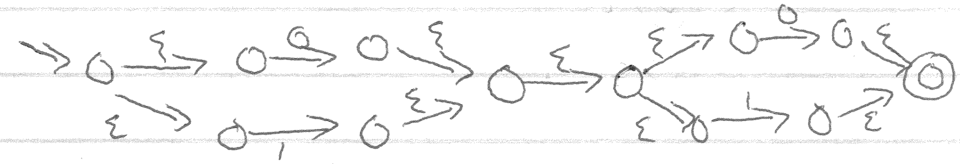
$C(\Sigma) = C(0 \cup 1)$

$C(0) =$ 

$C(1) =$ 

$C(\Sigma) =$  $=$ 

$C(\Sigma^*) =$ 

$C(\Sigma \circ \Sigma) =$ 

$\approx$ 

$\varepsilon \circ r = r = r \circ \varepsilon$ $\quad (\circ = \times,\ \varepsilon = 1,\ \emptyset = 0)$

$\emptyset \circ r = \emptyset = r \circ \emptyset$

$\emptyset \cup r = r = r \cup \emptyset$ $\quad (\cup = +,\ \emptyset = 0)$

$r_1 \cup r_2 = r_2 \cup r_1$

$\varepsilon^* = \varepsilon$ $\quad$ (* is like exponentiation)

$\emptyset^* = \varepsilon$

$r_1 \circ (r_2 \cup r_3) = r_1 \circ r_2 \cup r_1 \circ r_3$ $\quad$ ($\cup$ and $\circ$ distribute)

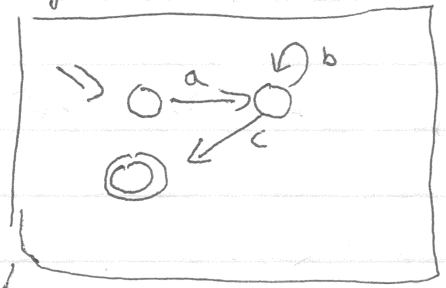
~~$(r_2 \cup r_3)^* = r_2^* \cup r_3^*$~~

$r_1^{RR} = r_1$ $\quad$ $\boxed{\sim / \cup / \cap \text{ obey de-morgan/etc rules}}$

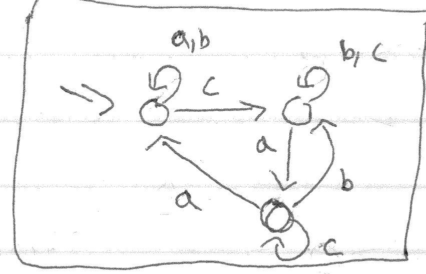
7-1/ CLAIM: $\forall D \in DFA.\ \exists R \in REX.\ L(D) = L(R)$

A constructive version is a dissambler = DIS

Input D: Output R:



$\Longrightarrow$ $a\,b^*\,c$



$(a \cup b)^* c\, (b \cup c)^* a$

??

DIS (n-state DFA d) = IN $\circ$ RIP^n $\circ$ OUT

IN (n-DFA $\Rightarrow$ n+2 - GNFA)

RIP (n+1-GNFA $\Rightarrow$ n-GNFA)

OUT (2-GNFA $\Rightarrow$ REX)

$\to$ NFA: $Q \times \Sigma_\varepsilon \to P(Q)$

$\to$ NFA has a set

GNFA = $\langle \Sigma, Q, q_s, \delta, q_e \rangle$ $\quad q_s \neq q_e$

start end

$\delta : (Q - \{q_e\}) \times (Q - \{q_s\}) \to REX$

if $\delta(q_i, q_j) = r$ and $w \in L(r)$

then the original NFA $q_i \overset{w}{\Rightarrow}{}^* q_j$

IN (d) = $\langle \Sigma, Q \cup \{q_s, q_e\}, q_s, \delta', q_e \rangle$

$\forall q_i, q_j \in Q.\ \delta'(q_i, q_j) = a$ s.t. $\delta(q_i, a) = q_j$

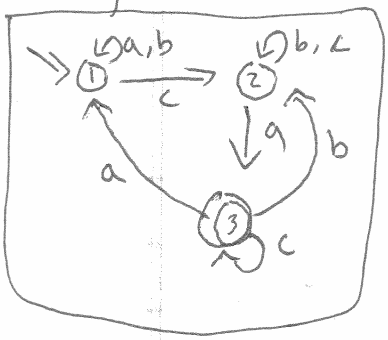
$(i) \xrightarrow{a} (j)$ $\quad (i,j) \to a$ $\qquad \emptyset$ if no-such-a exists

$\delta'(q_s, q_0) = \varepsilon$

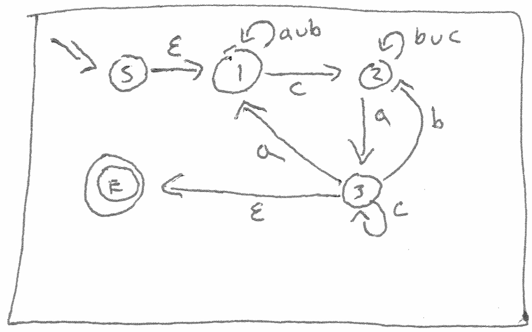
$\forall q_j \in F.\ \delta'(q_j, q_e) = \varepsilon$

7-2/

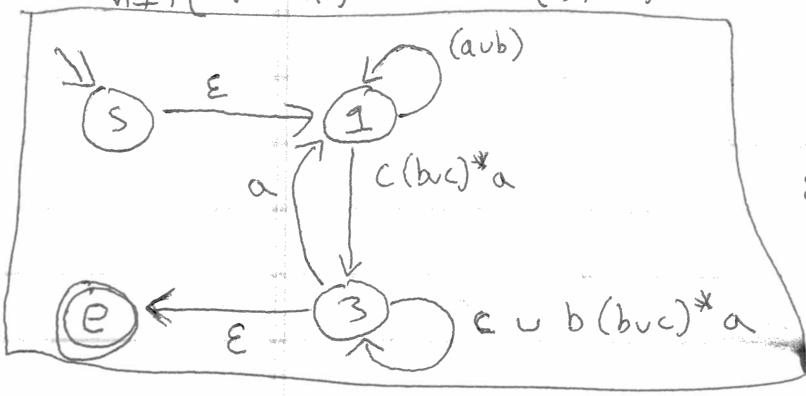
Original:



IN:

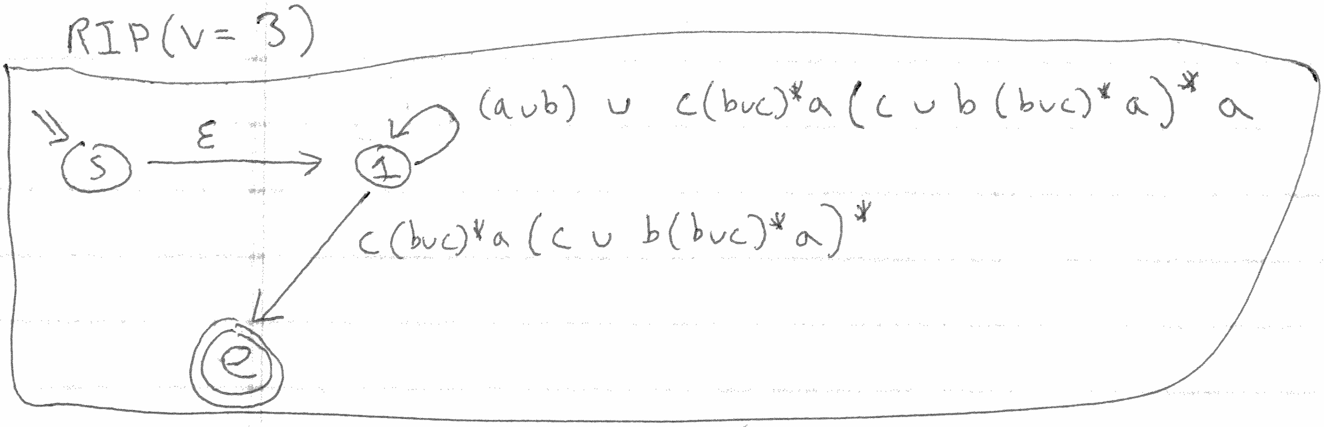


RIP(v = 2)

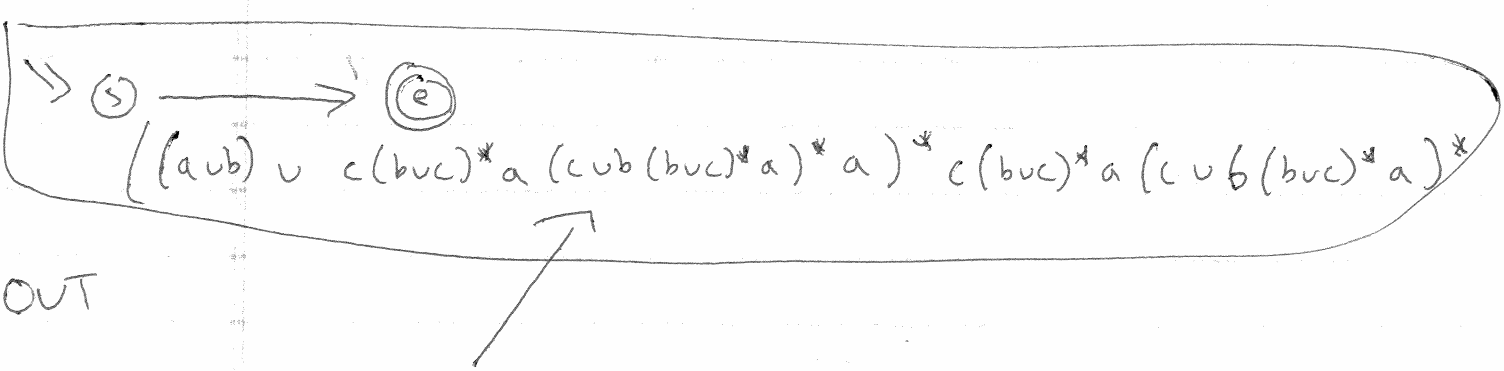


$$(s,1)' = (s,1) \cup (s,2)(2,2)^*(2,1)$$
$$\varepsilon \cup \emptyset \circ (b\cup c)^* \circ \emptyset$$
$$\varepsilon \cup \emptyset$$
$$\varepsilon$$

RIP(v = 3)

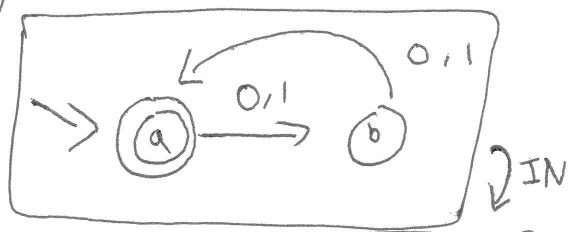


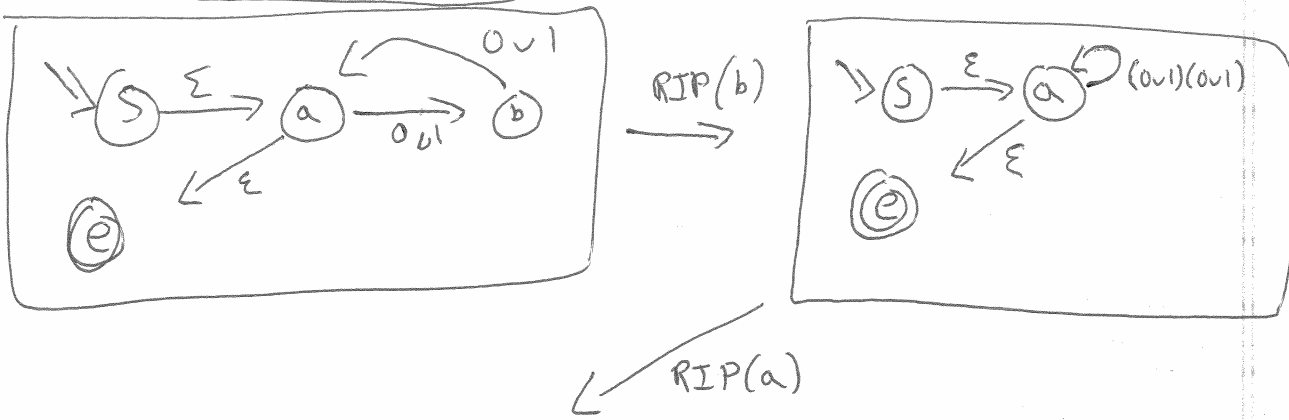
RIP(v = 1)

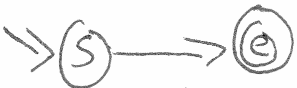


OUT

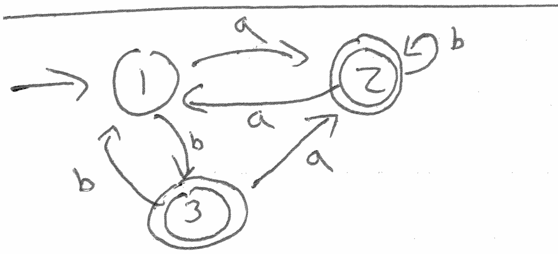
7-2b

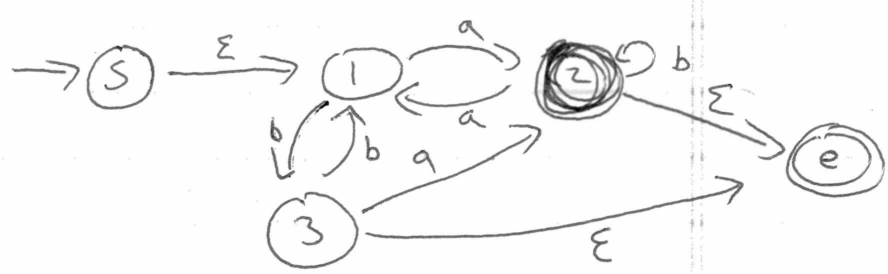


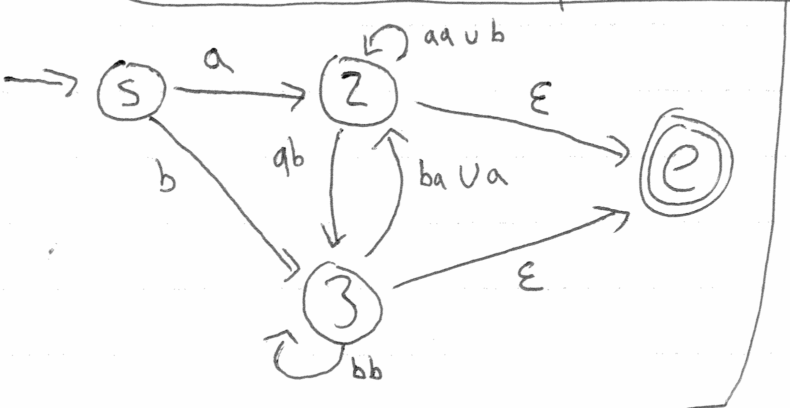


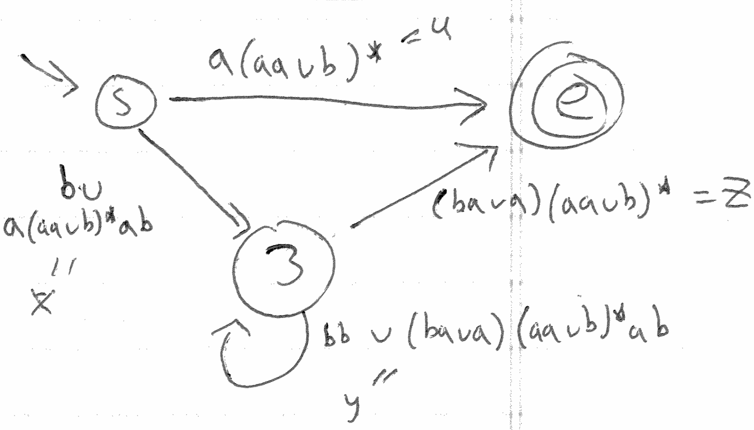


$\varepsilon((0\cup1)(0\cup1))^*\varepsilon \quad = \quad ((0\cup1)(0\cup1))^* \quad = \quad 2n$









$u \cup xy^*z$

7-3/ OUT(2-GWFA $\Rightarrow$ REX) = R

$\langle \Sigma, \{q_s, q_e\}, q_s, \{((q_s, q_e), R)\}, q_e \rangle$

$$\underbrace{Q - q_e}_{q_s} \times \underbrace{Q - q_s}_{q_e}$$

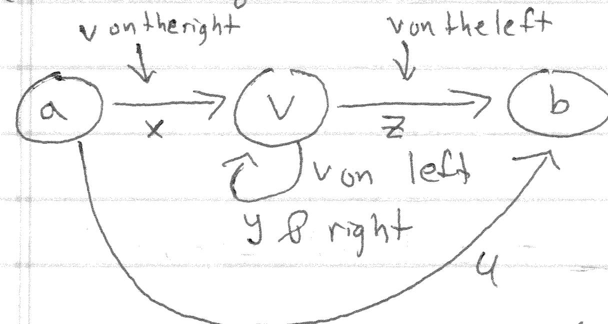
RIP(n+1 $\Rightarrow$ n)

in: $\langle \Sigma, Q \cup \{q_v\}, q_s, \delta, q_e \rangle$

out: $\langle \Sigma, Q, q_s, \delta', q_e \rangle$

$\delta: (Q \cup \{q_v\} - q_e \times Q \cup \{q_v\} - q_s) \Rightarrow$ REX

$\delta': (Q - q_e \times Q - q_s) \Rightarrow$ REX

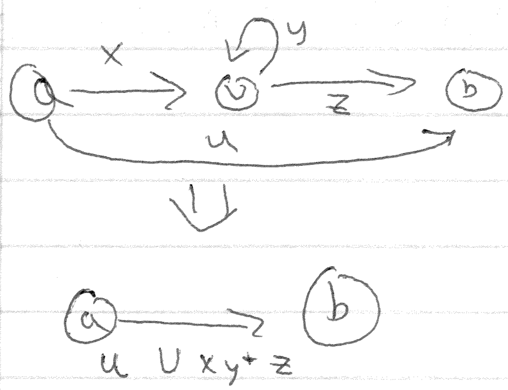


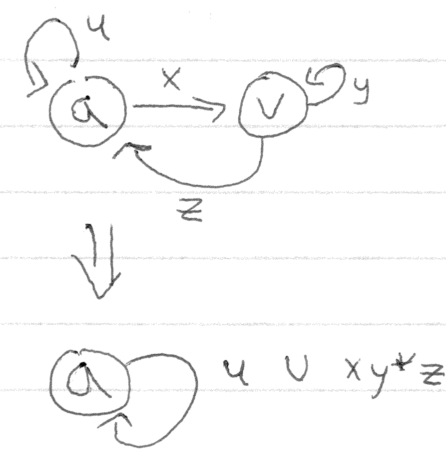
$\delta(a,b) = u$

$\delta'(a,b) = u \cup xy^*z$

Path from a to b = $\delta(a,b)$

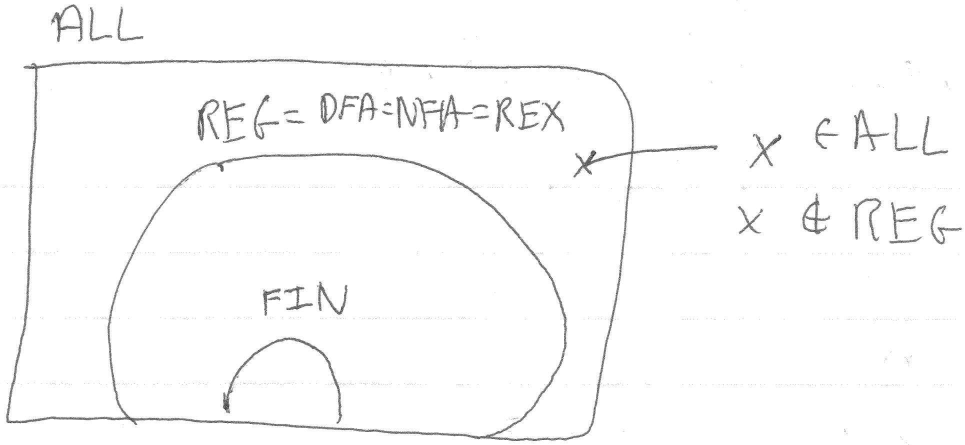
$$\delta'(q_a, q_b) = \delta(q_a, q_b) \cup \delta(q_a, q_v) \circ \delta(q_v, q_v)^* \circ \delta(q_v, q_b)$$





b=a

3-4



"x ∈ REG" a proof is a DFA, NFA, or REX = $\exists d \in DFA.\ L(d) = x$

what's a proof of "x ∉ REG"?

$\neg \exists d \in DFA.\ L(d) = x$

$= \forall d \in DFA.\ L(d) \neq x$

1) $\forall d \in DFA.\ F(L(d))$ — $\neg \exists d \in DFA.\ L(d) = x$

2) $\neg F(x)$

ALT. $\exists d \in DFA.\ L(d) = x$

do $\Rightarrow$ ① that $F(x)$ $\Rightarrow$ ② FALSE

What is F? Bad F: "A contains ε" ① isn't true

Bad F: "If $x \in A$, then $\exists A'.\ A = A' \cup \{x\}$" ② is never false