4-2

$L(\text{NFA } n) = \{ x \mid x \text{ is accepted by NFA } n \}$

A string x is accepted by NFA n iff

$$q_0 \overset{x}{\Rightarrow}^* q_i \quad \text{s.t.} \quad q_i \in F$$ [exactly the same as DFA

An NFA n runs from q_i to q_j on x $\left(q_i \overset{x}{\Rightarrow}^* q_j \right)$

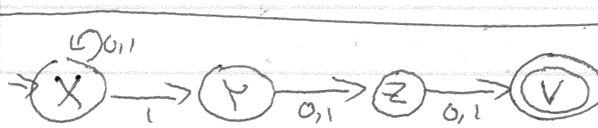
$q_i \overset{\epsilon}{\Rightarrow}^* q_i \qquad q_i \overset{ax}{\Rightarrow}^* q_k$ iff $q_i \overset{a}{\Rightarrow} q_j$ where $q_j \in Q$

$q_j \overset{x}{\Rightarrow}^* q_k \qquad a \in \boxed{\Sigma_\epsilon}$

only difference from DFA

An NFA n steps from q_i to q_j on $\underbrace{a}_{\Sigma_\epsilon}$ $\left(q_i \overset{a}{\Rightarrow} q_j \right)$ (for DFA $= \Sigma$)

$$q_j \underbrace{\in}_{= \text{ for DFA}} \delta(q_i, a)$$



$\overset{1}{\Rightarrow}$ [Y] 100 $\overset{1}{\Rightarrow}$ [Z] 00 $\overset{0}{\Rightarrow}$ [V] 0 0

[X] 11100 $\overset{1}{\Rightarrow}$ [X] 1100 $\Rightarrow$ [X] 100 $\Rightarrow$ [X] 00 $\Rightarrow$ [X] 0 $\Rightarrow$ [X] 0

$\Rightarrow$ [Y] 00 $\Rightarrow$ [Z] 0 $\Rightarrow$ [V

$\Rightarrow$ [Y] 1100 $\Rightarrow$ [Z] 100 $\Rightarrow$ [V] 00

$Y \in \delta(X, 1) = \{X, Y\}$

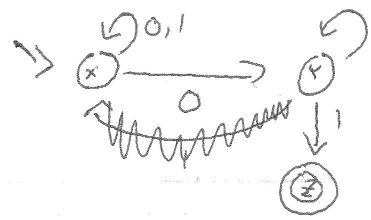
$$X \overset{11100}{\Rightarrow} V \ ?\checkmark \qquad 11100 \in L(n)$$

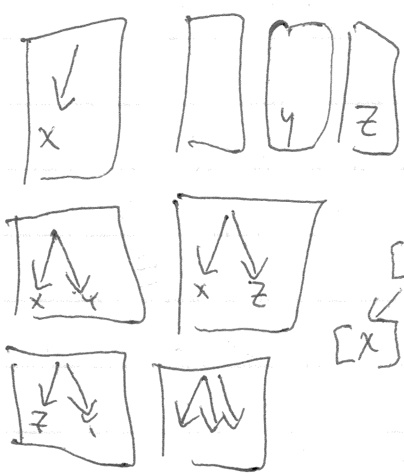
back-tracking = DFS   oracle = cheating

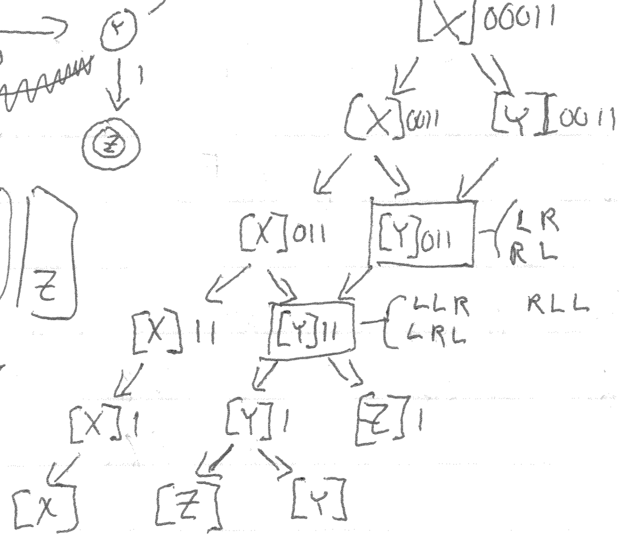
fork()-ing = BFS

1-3/

00011







$[\{X\}]$ 00011

$\downarrow$

$[\{X, Y\}]$ 0011

$\downarrow$

$[\{X, Y\}]$ 011

$\downarrow$

$[\{X, Y\}]$ 11

$\downarrow$

$[\{X, Y, Z\}]$ 1

$\downarrow$

$[\{X, Y, Z\}]$ ✓

5-1/ Claim: DFAs & NFAs are equally powerful

$FIN \subseteq REG \;::\; \forall A \in FIN.\ \exists D \in DFA.\ L(D) = A$

(1) $\forall N \in NFA.\ \exists D \in DFA.\ L(N) = L(D)$ $[NFA \subseteq DFA]$

(2) $\forall D \in DFA.\ \exists N \in NFA.\ L(D) = L(N)$ $[DFA \subseteq NFA]$

(2) In: $\langle \Sigma, Q, q_0, \delta, F \rangle = d$

Out: $n = \langle \Sigma, Q, q_0, \delta', F \rangle$

$\delta : Q \times \Sigma \to Q \qquad \delta' : Q \times \Sigma \to P(Q)$

$$\delta'(q, s) = \{\delta(q, s)\}$$

1) In: $N = \langle \Sigma, Q, q_0, \delta, F \rangle$

Out: $d = \langle \Sigma, Q', q_0', \delta', F' \rangle$

$Q' = P(Q)$ // size of d is exponential in n

$q_0' = \{q_0\}$

$F' = \{x \mid x \in Q' \text{ and } x \cap F \neq \emptyset\}$

$\delta' : (q : P(Q))\ (s : \Sigma) \to (a : P(Q))$

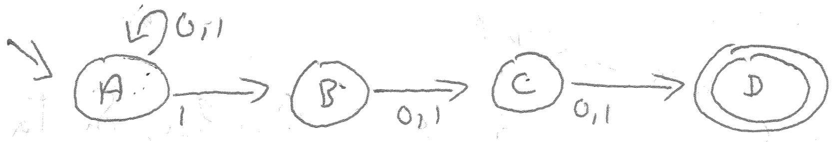
$a = \bigcup_{q \in pq} \delta(q, s)$

NFA

$\delta : Q \times \Sigma \to P(Q)$

DFA $\to \delta' : Q' \times \Sigma \to Q'$

-2/

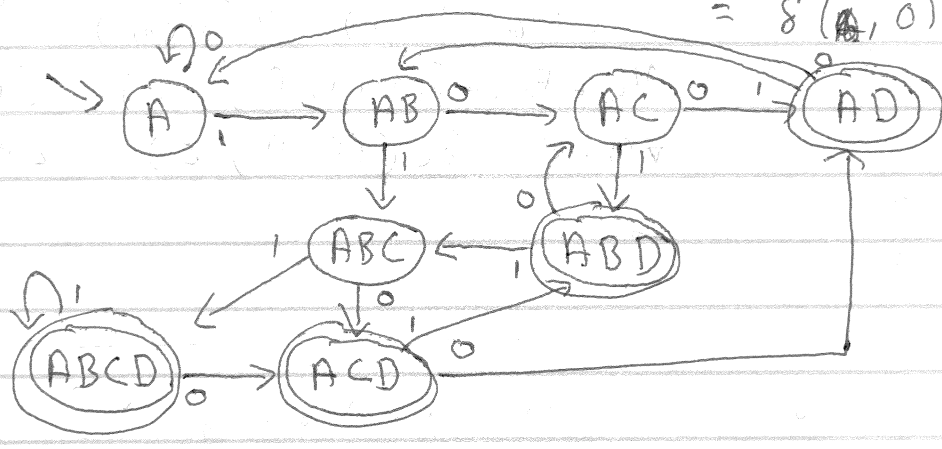


$Q = \{A, B, C, D\}$

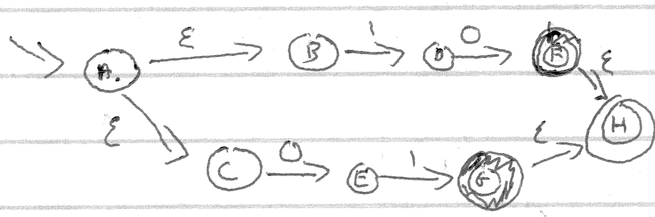
$Q' = P(Q) = \{\}$ A B C D AB AC AD BC BD CD ABC ABD BCD ACD ABCD

$q_0' = \{A\}$ $\delta'(\{A\}, 0) = \bigcup_{q \in \{A\}} \delta(q, 0)$

$= \delta(A, 0) = \{A\}$



D = 1 3 ago
C = 1 2 ago
B = 1 1 ago



BEFORE: $q_0' = \{q_0\}$

$\delta'(pq, s) = \bigcup_{q \in pq} \delta(q, s)$

$E(\{a\}) = \{a, b, c\}$

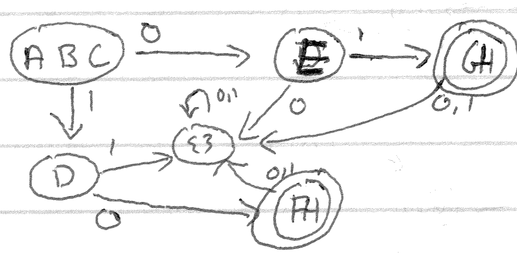
$q_0' = \{A, B, C\}$ $q_0' = E(\{q_0\})$

$E : P(Q) \to P(Q)$

$E(pq) = E(pq \cup \bigcup_{q \in pq} \delta(q, \varepsilon))$

compute least-fixed point

$\delta'(pq, s) = E(\bigcup_{q \in pq} \delta(q, s))$



5-3/ Star, Reversal, Concatenate

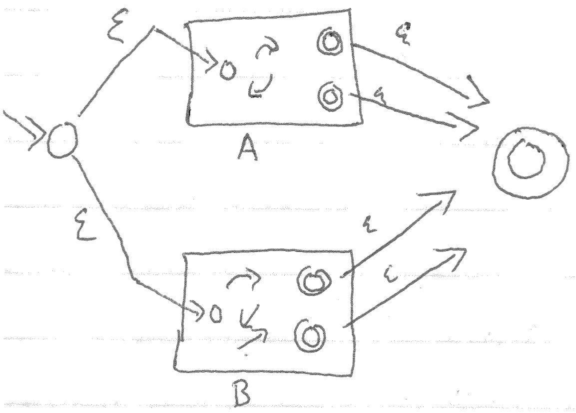
$\{a,b\}$ $\quad 1 \times Q_a = (1,a)\ (1,b)$

$$\text{Union}(\overset{:DFA}{A}, \overset{:DFA}{B}) = \overset{:NFA}{C} \rightarrow C' : DFA$$

$$A = \langle \Sigma, Q_a, q_{0a}, \delta_a, F_a \rangle$$

$Q_b \times 1 = (a,1)(b,1)$

$$B = \langle \Sigma, \overset{=\{a,b\}}{Q_b}, q_{0b}, \delta_b, F_b \rangle$$



$$C = \langle \Sigma, \overset{Q_c =}{\{\text{start}, \text{end}\}} \cup (1 \times Q_a) \cup (Q_b \times 1),$$

$q_{0c} = \text{start}$, $\quad F_c = \{\text{end}\}$,

$\delta_c(\text{start}, \varepsilon) = \{(1, q_{0a})\ (q_{0b}, 1)\}$

$\delta_c((1, q_a), s) = \{(1, \delta_a(q_a, s))\}$

$\delta_c((q_b, 1), s) = \{(\delta_b(q_b, s), 1)\}$

$\forall f_a \in F_a$, $\delta_c((1, f_a), \varepsilon) = \{\text{end}\}$

$\forall f_b \in F_b$. $\delta_c((f_b, 1), \varepsilon) = \{\text{end}\}$

OLD-union $= A \times B$

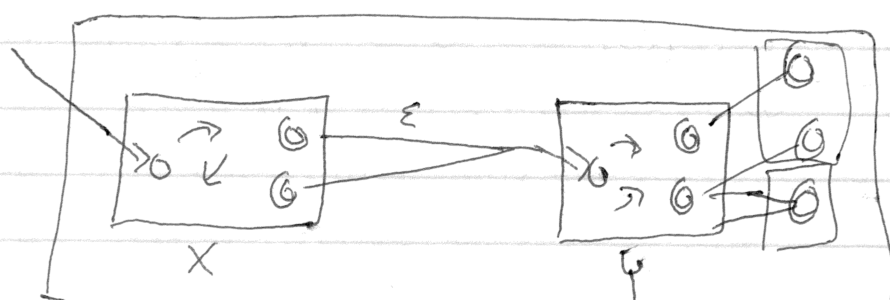
NEW-union $= A + B + 2$

DFA-ver $= 2^{(A+B+2)}$

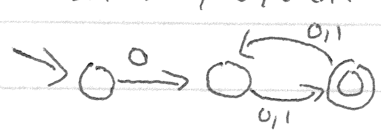
Concatenate : $X \circ Y = \{ xy \mid x \in X \text{ and } y \in Y \}$

$\{a,b\} \circ \{c,d\} = \{ac, ad, bc, bd\}$

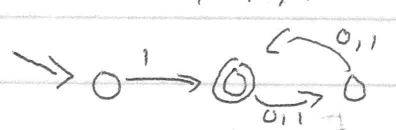
\# {starts w/ 0, even} ∘ {starts w/ 1, odd} = { an odd string starting w/ 0 with an even prefix, followed by 1 and then the rest }



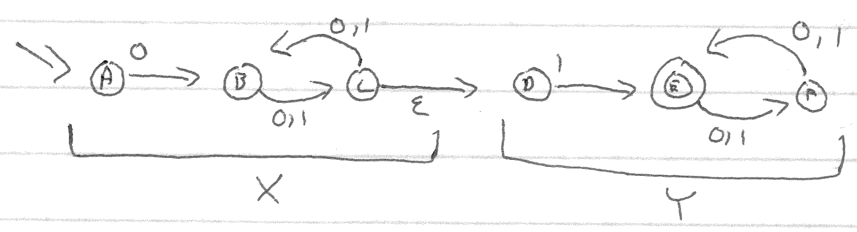
starts w/ 0, even:



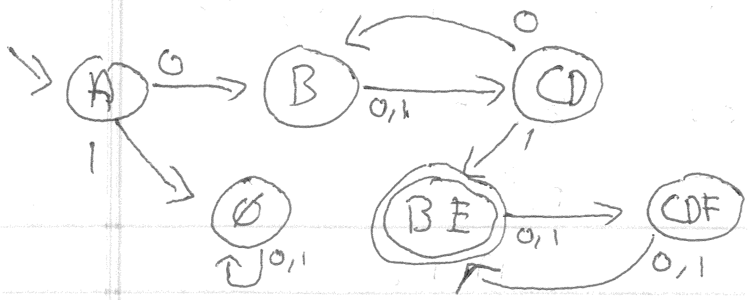
starts w/ 1, odd:



concatenate:

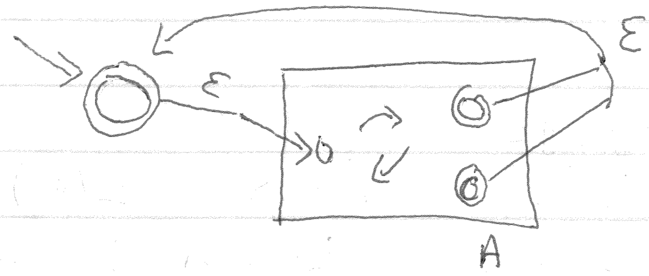


5-4



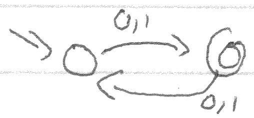
Star : $A^* = \{ \cancel{\ } \; x_1 \ldots x_n \mid x_i \in A \}$ $n \in [0, \ldots)$

$\varepsilon \in A^*$ $A^* = \varepsilon \cup A \circ A^*$

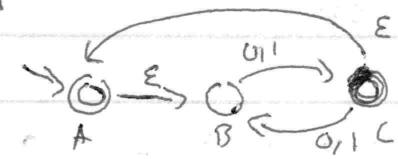


$2n+1$
$2m+1$ $> 2(n+m+1$

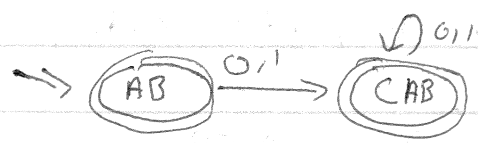
$A =$ odd



A^*

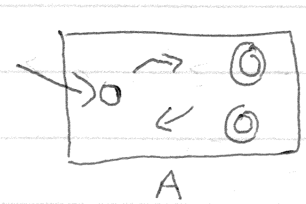


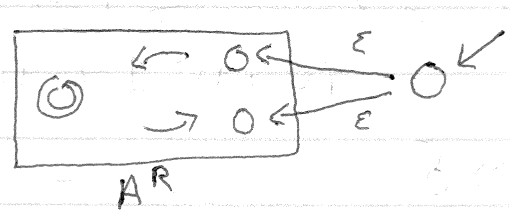
$C(A^*)$

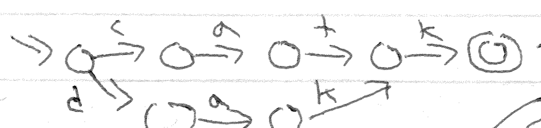


Reversal $A^R = \{ x \mid x^R \in A \}$

$A = \{ cat \}$ $A^R = \{ tac \}$

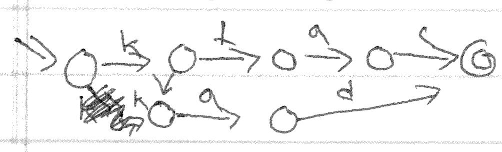






– {catk, dakk}

{kkad, ktac}



1-2/ Regular Operations: $\cup, \circ, *, R, \cap, \sim$

Regular Expression re over Σ =

$\emptyset$ $\quad$ r_1^R

ε $\quad$ $r_1 \cap r_2$

$a \in \Sigma$ $\quad$ $\sim r_1$

$r_1 \cup r_2$ // $(r_1 | r_2)$ $\quad$ $\bullet$ $\quad$ $L(\bullet) = \Sigma$

$r_1 \circ r_2$ // $r_1 r_2$

r_1^*

L : regexp $\Rightarrow$ language

$L(\emptyset) = \emptyset$ $\quad$ $L(\varepsilon) = \{\varepsilon\}$ $\quad$ $L(a) = \{a\}$

$L(r_1 \cup r_2) = L(r_1) \cup L(r_2)$ $\quad$ $L(r_1 \circ r_2) = L(r_1) \circ L(r_2)$

and so on...

$L(cat) = \{cat\} = L(c \circ a \circ t)$

$L(c(a \cup d)^* r) = \{cr, car, cdr, cadr, cdar, caaaadr, \ldots\}$

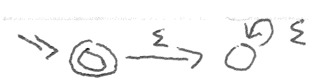
REX = REG? $\quad$ (DFA = NFA)

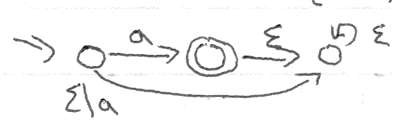
① $\forall R \in REX. \exists D \in DFA. L(R) = L(D)$ $\longleftarrow$ compiler from regexp to DFAs – C

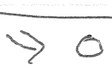
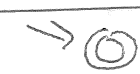
② $\forall D \in DFA. \exists R \in REX. L(D) = L(R)$ $\longleftarrow$ dissassembler – D

D([3 from end]) = $\quad$ C( ) = 1 from end

$(0 \cup 1)^* 1 (0 \cup 1)(0 \cup 1) = \Sigma^* 1 \Sigma \Sigma = \bullet^* 1 \bullet \bullet$

C($\emptyset$) =  $\quad$ C(ε) = 

C(a) = 

C($\emptyset$) =  $\quad$ C(ε) =  $\quad$ C(a) =