~~Lecture~~ 24-1

$TM \simeq \Sigma_0 \simeq \Sigma_1 \simeq N$

$B \not\simeq N$

$ALL = P(\Sigma^*)$ = all sets of strings of Σ

= all languages

$\{0, 1\} \in ALL$ $\qquad$ $\{0^n 1^n\} \in ALL$

$\{\varepsilon, 00, 11, 001\} \in ALL$

Every string from Σ^* appears in some order lexicographically

$\varepsilon, 0, 1, 00, 01, 10, 11, 000, 001, \ldots$

$\downarrow\ \Sigma^* \downarrow \simeq \downarrow N \downarrow\ \ \downarrow\ \ \downarrow\ \ \downarrow$

$0\ \ 1\ \ 2\ \ 3\ \ 4\ \ 5\ \ 6$ $\qquad$ $\underbrace{\Sigma^* \simeq N}_{\to \ell}$

$ALL \not\simeq N$ because $ALL \simeq B$

$\exists m : ALL \to B.\ (\forall a, b \in ALL.\ m(a) = m(b) \Rightarrow a = b)$

$\wedge\ (\forall b \in B, \exists a \in ALL\ m(a) = b)$

$m(s) = (\text{fun } i \Rightarrow \text{if } \ell(i) \in s \text{ then } 1 \text{ ow } 0)$

$m(\{0, 1\}) = 011\overline{0}$

$m(\{\varepsilon, 00, 11, 001\}) = 100100101\overline{0}$

each digit of $m(s)$ says if a string is in the language

$TM \simeq \Sigma_0 \simeq \Sigma_1 \simeq N$

$N \not\simeq B \simeq ALL$

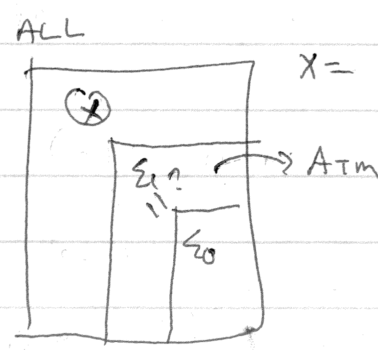
$\Sigma_1 \not\simeq ALL$ but $\Sigma_1 \subseteq ALL$

therefore ALL is bigger

$\boxed{\exists x \in ALL.\ x \notin \Sigma_1.}$

$A_{TM} = \{\langle M, w\rangle \mid M \text{ accepts } w\}$

$\overline{A_{TM}} = \{\langle M, w\rangle \mid M \text{ does not accept } w, \text{ i.e. } M \text{ diverges on } w\}$



$x = \overline{A_{TM}}$

24-2

Assume that $A_{TM} \in \Sigma_0$. Call the machine H.

$$H(\langle M, w\rangle) = \begin{cases} \text{accept} & \text{if } M \text{ accepts } w \\ \text{reject} & \text{if } M \text{ does not accept } w \\ & \text{(rejects, diverges)} \end{cases}$$

OK

$D(\langle M\rangle)$ = " On input $\langle M\rangle$ where M is a TM,
1. Run H on $\langle M, \langle M\rangle\rangle$
2. output the opposite of H "

$$= \begin{cases} \text{accept} & \text{if } M \text{ does not accept } \langle M\rangle \\ \text{reject} & \text{if } M \text{ accepts } \langle M\rangle \end{cases}$$

Run $D(\langle D\rangle) = \begin{cases} \text{accept if} & D \text{ does not accept } \langle D\rangle \\ \text{reject if} & D \text{ accepts } \langle D\rangle \end{cases}$

Thus $D(\langle D\rangle)$ can't return
then it must diverge
then it is not in Σ_0
then H must diverge
so $H \notin \Sigma_0$
therefore contradiction and $A_{TM} \notin \Sigma_0$

"Liar's Paradox":
"This sentence is false."

$A_{TM} \in \Sigma_1$ and $A_{TM} \notin \Sigma_0$

Claim: $X \in \Sigma_0$ iff $X \in \Sigma_1$ and $\overline{X} \in \Sigma_1$

$\Rightarrow$) given $X \in \Sigma_0$. $X \in \Sigma_1$ is obvs. $\overline{X} \in \Sigma_1$ by flipping result

$\Leftarrow$) On input w,
run $X(w)$ and $\overline{X}(w)$ in parallel

$X(w) \Downarrow$ accept, $\overline{X}(w) \Downarrow$ reject

---

$\neg X \in \Sigma_0$ iff $X \notin \Sigma_1$ or $\overline{X} \notin \Sigma_1$ $\Rightarrow$ $\overline{A_{TM}} \notin \Sigma_1$