19-1

Normal TM

$\delta : Q \times \Gamma \to Q \times \Gamma \times \{L, R\}$

current state, symbol read, next state, symbol written, direction

$$\frac{\delta(q_i, b) = (q_j, c, R)}{ua[q_i]bv \Rightarrow uac[q_j]v} \qquad \frac{\delta(q_i, b) = (q_j, c, L)}{ua[q_i]bv \Rightarrow u[q_j]acv}$$

~~Skip~~ TMs
Don't write

$\delta : Q \times \Gamma \to (Q \times \Gamma \times \{L, R\}) + (Q \times \{L, R\})$

$$\frac{\delta(q_i, b) = (q_j, R)}{ua[q_i]bv \Rightarrow uab[q_j]v} \qquad \frac{\delta(q_i, b) = (q_j, L)}{ua[q_i]bv \Rightarrow u[q_j]abv}$$

$$\delta'(q_i, b) = \begin{cases} (q_j, c, d) & \text{if } \delta(q_i, b) = (q_j, c, d) \\ (q_j, b, d) & \text{if } \delta(q_i, b) = (q_j, d) \end{cases}$$

~~Don't read~~ TM
Skip

$\delta : (\underbrace{Q \times \Gamma \to (Q \times \Gamma \times \{L, R\})}_{\text{normal}}) + (Q \to (\underbrace{Q \times \{L, R\}}_{\text{don't read or write}}))$

$\forall g \in \Gamma, \ \delta(q_i, g) = (q_j, g, d)$

Stay TM

$\delta : Q \times \Gamma \to Q \times \Gamma \times \{L, R, S\}$

$$\frac{\delta(q_i, b) = (q_j, c, S)}{ua[q_i]bv \Rightarrow ua[q_j]cv}$$

$\forall d \in \Gamma,$

$$\frac{\delta(q_i, b) = (q_*, c, R)}{} \qquad \frac{\delta(q_*, d) = (q_j, d, L)}{}$$

$ua[q_i]bv \Rightarrow uac[q_*]v \Rightarrow ua[q_j]cv$

$v = dv'$

1 use of S $\Rightarrow$ 1 state w/ $|\Gamma|$ rules

1 step $\Rightarrow$ 2 steps

$$\frac{\delta(q_i, b) = (q_j, c, S)}{} \qquad \frac{\delta(q_j, c) = (\overset{q_k}{?}, \overset{d}{?}, \overset{R}{?})}{}$$

$ua[q_i]bv \Rightarrow ua[q_j]cv \Rightarrow uad[q_k]v$

$\delta(q_i, b) = (q_k, d, R) \qquad \Rightarrow u[q_k]adv \quad (\text{if } L)$

19-2) Stay = Normal

means

$L(sm) = L(c(sm))$

$$\begin{tikzcd}
sm\ \ c_0 \arrow[r, "compile"] \arrow[d, Rightarrow, "step"] & c(sm)\ \ c'_0 \arrow[d, Rightarrow, "step"] \\
sm\ \ c_1 & c'_1 \arrow[d, Rightarrow, "step"] \\
 & c'_2 \arrow[ul, "decompile"]
\end{tikzcd}$$

$$\begin{tikzcd}
ua[q_i]bv \arrow[r, "compile"] \arrow[d, Rightarrow, "stay-step"] & ua[\hat{q}_i]bv \arrow[d, Rightarrow, "normal-step"] \\
ua[q_j]cv & uac[q_*]v \arrow[d, Rightarrow, "normal-step"] \\
 & ua[\hat{q}_j]cv \arrow[ul, "decompile"]
\end{tikzcd}$$

a simulation theorem
(bi-)

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reverse-TM on input w, the initial ~~tape~~ config is $w^R[q_0]$

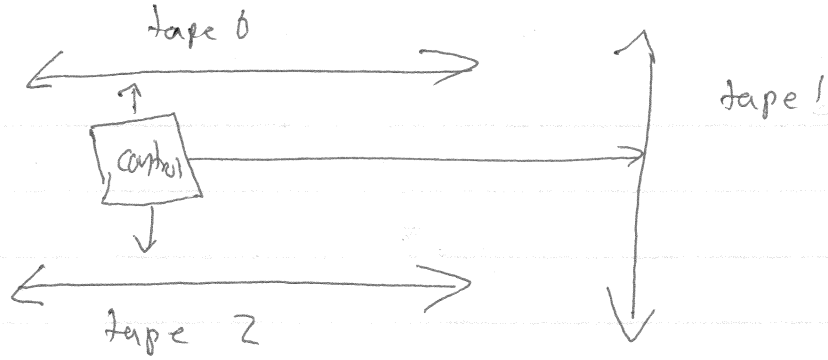
L means R / vice versa

compile $(u[q_i]v) = (v^R[\hat{q}_i]u^R)$

$\delta(q_i, b) = (q_j, c, L) \Rightarrow \delta(\hat{q}_i, b) = (\hat{q}_j, c, R)$

19-3

Multi-tape Turing Machine (k-tapes)



k-config: $\begin{matrix} u_0 \\ \vdots \\ u_k \end{matrix} \left[q \right] \begin{matrix} v_0 \\ \vdots \\ v_k \end{matrix}$ $\qquad \delta : Q \times \Gamma^k \Rightarrow Q \times (\Gamma \times \{L, R\})^k$

$$\frac{\delta(q_i, b_0, b_1) = (q_j, c_0, L, c_1, R)}{\begin{matrix} u_0 a_0 \\ u_1 a_1 \end{matrix} \left[q_i \right] \begin{matrix} b_0 v_0 \\ b_1 v_1 \end{matrix} \Rightarrow \begin{matrix} u_0 \\ u_1 a_1 c_1 \end{matrix} \left[q_j \right] \begin{matrix} a_0 c_0 v_0 \\ v_1 \end{matrix}}$$

$u^k [q_i] v^k \Rightarrow u'^k [q_j] v'^k \qquad b^k = v_0^k$

$\delta(q_i, b^k) = (q_j, (c, d)^k)$

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k=2 $\quad \begin{matrix} abc \\ uvw \end{matrix} \left[q_i \right] \begin{matrix} def \\ xyz \end{matrix} \xrightarrow{\text{compile}} [\hat{q}_i]\, abc\,\hat{d}ef \blacksquare uvw\hat{x}yz$

$\Downarrow$

$a [\hat{q}_i/fst]\, bc\hat{d}ef \blacksquare uvw\hat{x}yz$

$\Downarrow$

$abc\hat{d} [\hat{q}_i/d]\, ef \blacksquare uvw\hat{x}yz$

$\begin{matrix} ab \\ uvwx' \end{matrix} \left[q_j \right] \begin{matrix} cd'ef \\ yz \end{matrix}$

$abc\hat{d}ef \blacksquare [\hat{q}_i/d/snd]\, uvw\hat{x}yz$

$abc\hat{d}ef \blacksquare uvw\hat{x} [\hat{q}_i/d/x]\, yz$

1 step $\Rightarrow$ $2 \times |\text{tape}|$

$abc\hat{d}ef \blacksquare uvw\hat{x} [\hat{q}_j/d'R/x'L]\, yz$

1 rules $\Rightarrow$ $3 \times k \times |\Gamma|^k$

$abc\hat{d}ef [\hat{q}_j/d'R] \blacksquare uvwx'\hat{y}z$

$[\hat{q}_j/\text{done}]\, ab\hat{c}d'ef \blacksquare uvwx'\hat{y}z$

$[\hat{q}_j]\, ab\hat{c}d'ef \blacksquare uvwx'\hat{y}z$

19-4

$$\delta(q_i, b, d) = (q_i, b', R, d', R)$$

$$\begin{matrix} u\,a \\ x\,c \end{matrix} \begin{bmatrix} q_i \\ \\ \end{bmatrix} \begin{matrix} b \\ d\,y \end{matrix} \implies \begin{matrix} u\,a\,b' \\ x\,c\,d' \end{matrix} \begin{bmatrix} q_i \\ \\ \end{bmatrix} \begin{matrix} \sqcup \\ y \end{matrix}$$

how does the blank get put in?

$u a \hat{b} \blacksquare x c \hat{d} y$ $\qquad$ $u a b' \hat{\sqcup} \blacksquare x c d' \hat{y}$

after knowing what to write, while moving left, we must add a blank before/after the $\blacksquare$s

$u a \hat{b} \blacksquare x c \hat{d} y$

$u a b' \hat{\sqcup} \blacksquare x c d' \hat{y}$

~~multi-state~~

k-tape - TM

$\Downarrow$

Insert-TM

$\Downarrow$

normal-TM

Insert-TM

$\delta : Q \times \Gamma \Rightarrow Q \times \Gamma \times \{L, R, I\}$

$$\frac{\delta(q_i, b) = (q_i, c, I)}{ua[q_i]bv \Rightarrow ua[q_i]cbv}$$