18-4 / $Q = q_0 \cdots q_{14}$ $\Sigma = \{0,1\}$ $\Gamma = \{0,1,\sqcup\}$

$\Sigma = \{00, 01, 10, 00$ $\left\{\begin{matrix}0\\0\end{matrix}, \begin{matrix}0\\1\end{matrix}, \begin{matrix}1\\0\end{matrix}, \begin{matrix}0\\0\end{matrix}\right\}$ $\Gamma = \{0,1,\sqcup\}$

to add "x+y" where $x = x_n x_{n-1} \cdots x_2 x_1 x_0$

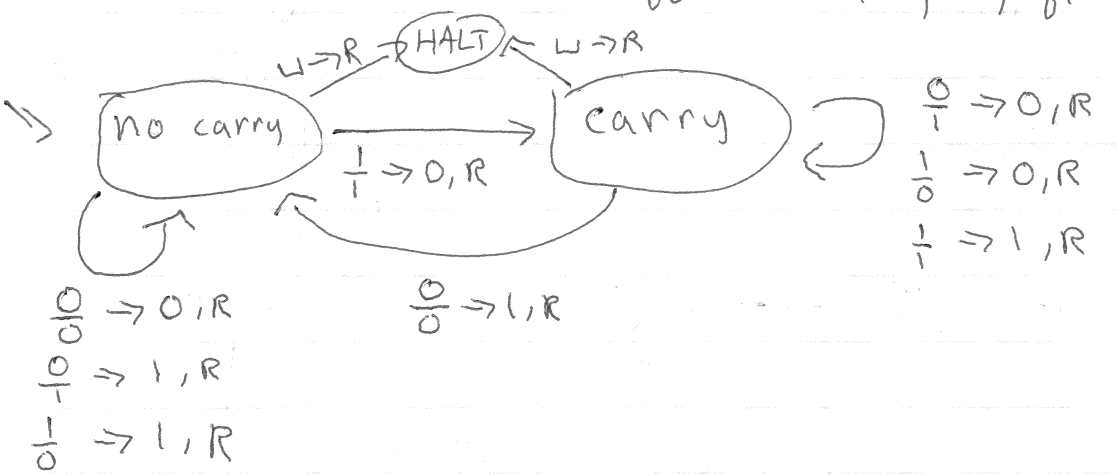
then input $\begin{matrix}x_0 & x_1 & \cdots & x_n\\ y_0 & y_1 & & y_n\end{matrix}$

$0110 + 0101 \Rightarrow \begin{matrix}0&1&1&0\\1&0&1&0\end{matrix}$

$[q_0]\ \begin{matrix}0&1&1&0\\1&0&1&0\end{matrix} \Rightarrow 1\,[q_0]\ \begin{matrix}1&0\\0&1\end{matrix} \Rightarrow 11\,[q_0]\ \begin{matrix}1&0\\1&0\end{matrix}$

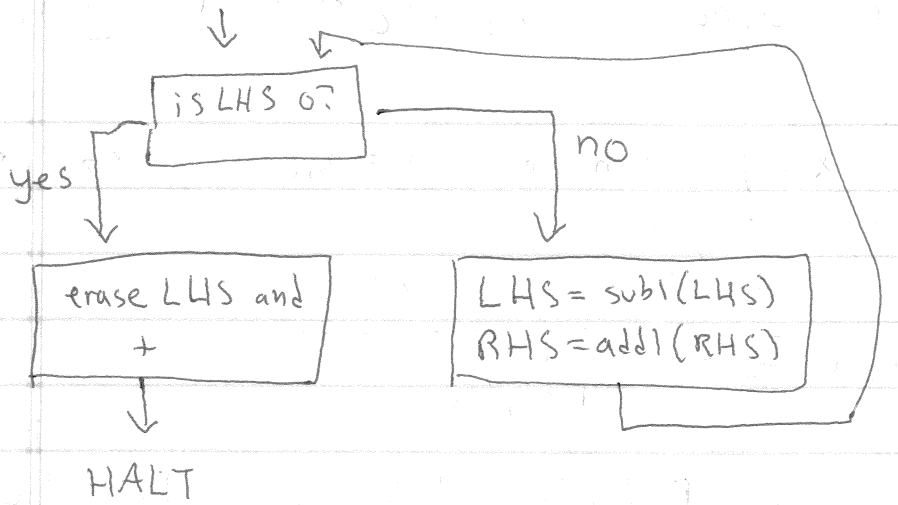
$\Rightarrow 110\,[q_1]\ \begin{matrix}0\\0\end{matrix} \Rightarrow 1101\,[q_0] \Rightarrow 1101\,[\text{HALT}]$

answer: 1011 q_0 = no carry, q_1 = carry



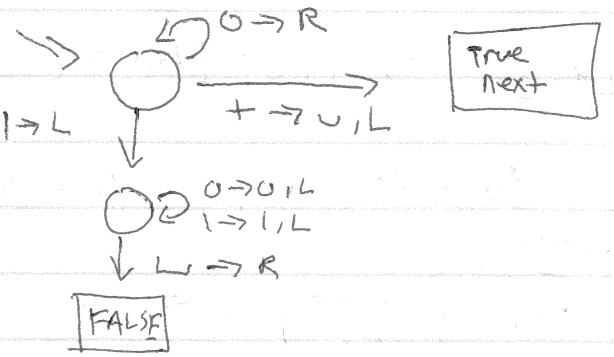
18-3

0110 + 0101 ⇒ 1011

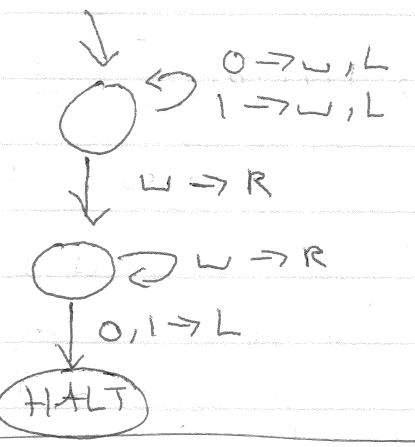


14 states

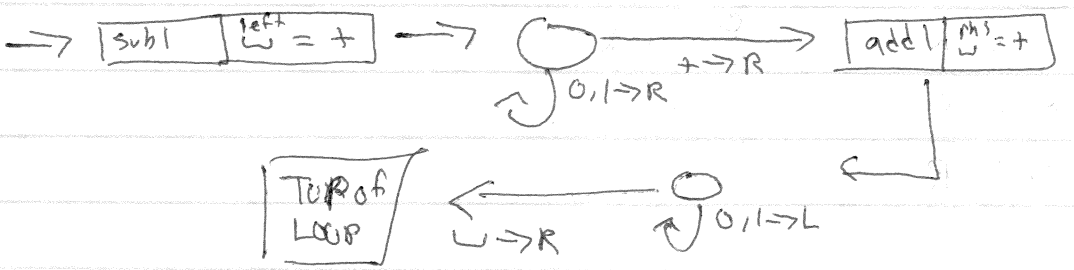
Zero?:



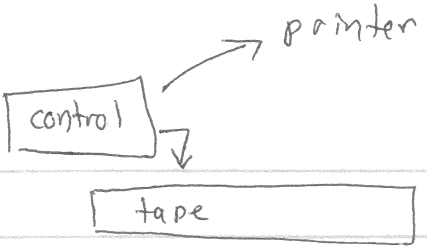
erase



sub1 / add1



Enumerator



$(Q, \Gamma, \delta, q_0, q_p)$

Tape is initially empty

every time we goto q_p, tape is printed

Transducer (when we go to q_a, return the tape)

add1 : 0010 ⇒ 0011

0011 ⇒ 0100

1111 ⇒ 10000

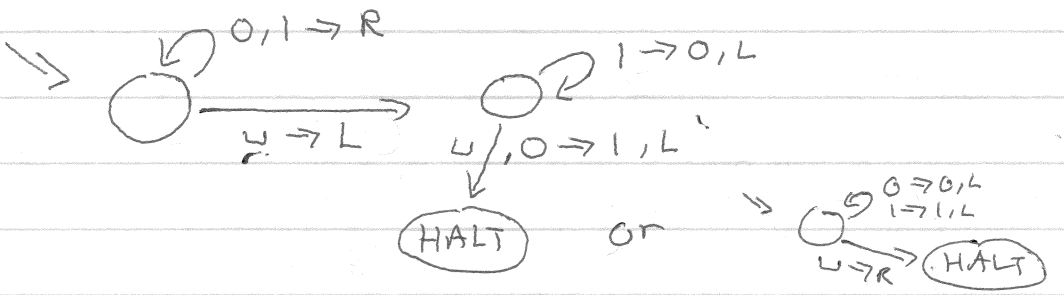
in ⇒ out, dir

in ⇒ dir

:= in ⇒ in, dir

in1, in2 ⇒ out, dir

in1, in2 ⇒ dir



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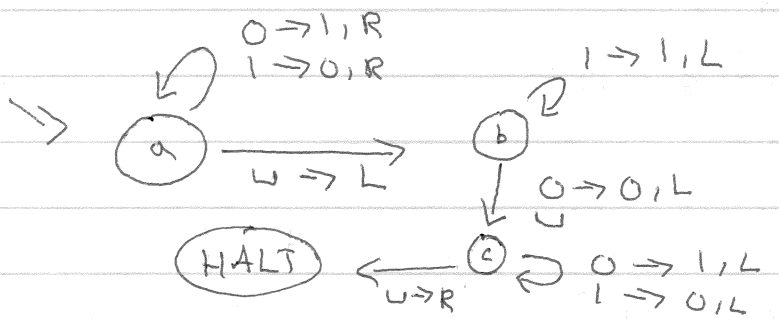
sub1

(2n)



[a] 0011 ⇒ 1[a]011 ⇒ 11[a]11

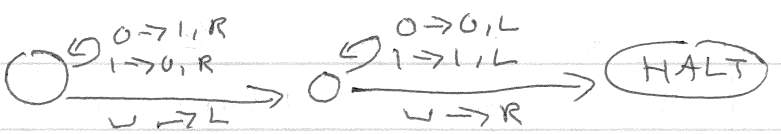
⇒ 110[a]1 ⇒ 1100[a] ⇒ 110[b]0

⇒ 11[c]00 ⇒ 1[c]110 ⇒ [c]1010

⇒ [c]␣0010 ⇒ [HALT]0010

sub1 = inv ∘ add1 ∘ inv (6n)

inv =



A Turing machine has 3 possibilites

1 ACCEPT

2 REJECT

3 LOOP (diverge)

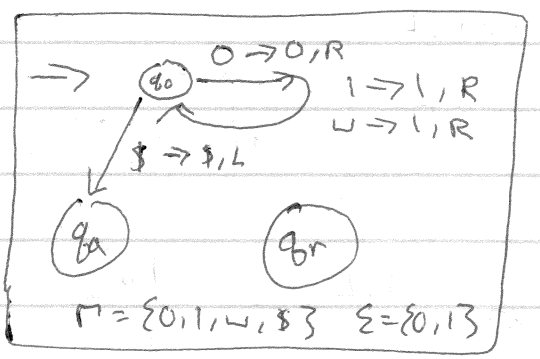
configuration

① $0 \to 0 \to (q_a)$ $\quad [q_0]u \Rightarrow^* x[q_a]y$

② $0 \to 0 \to (q_r)$ $\quad [q_0]u \Rightarrow^* x[q_a]y$

③  $\quad [q_0]u \Rightarrow^* x[q_i]y \Rightarrow^* x[q_i]y$

$0 \to 0 \to 0 \to \cdots \to 0 \to \cdots$



$[q_0]0110 \to 011[q_0]0$

$0[q_0]110 \to 0110[q_0]$

$01[q_0]10 \to 01101[q_0]$

$0110111[q_0]$

A recognizer accepts some input (all TMs are recognizers)

A decider never LOOPS (always ACCEPT or REJECT)

Turing-recognizable languages $= \Sigma_1$

$= \{ A \mid \exists m \in \text{recognizers}, L(m) = A \}$

Turing-decidable languages $= \Sigma_0$

$= \{ A \mid \exists m \in \text{deciders}, L(m) = A \}$

$\Sigma_0 \in \Sigma_1$