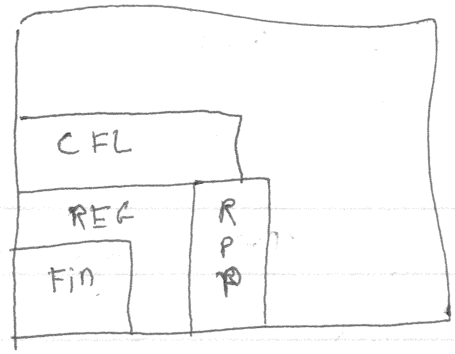
15-1



ALL = CFL?

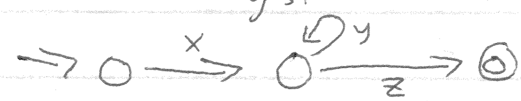
① $\forall A \in REG.\ F(A).$ ①+② ⇒ REG ≠ ALL

② $\exists A \in ALL.\ \neg F(A).$

F = Regular Pumping Property

If a string is long, then it visits many states, but there are only so many states, so it must repeat one, therefore it is a "recipe" for an infinite number of strings.

$s = xyz$



CPP – Context-free pumping property

A CFG has a limited number of variables (assume it's in Chomsky Normal Form)

↳ $A \to a$ $A \to BC$ $S \to \varepsilon$

A string s is in language and $|s| = 0$

$S \Rightarrow \varepsilon$

$|s| = 1$, $S \Rightarrow a$

$|s| = 2$, $S \Rightarrow BC \Rightarrow bc$ (R = $S \Rightarrow BC$, $B \Rightarrow b$, $C \Rightarrow c$)

$|s| = 3$, $S \Rightarrow BC \Rightarrow BDE$

DEC → bde, dec

$|s| = 4$,

$S \Rightarrow BC \Rightarrow BDE \Rightarrow BDFG$ (2, 3)

BFGE FGDE

A tree has at most 2^h leaves

A string of 2^{h+1} characters has at least h height

If $h = |V|$, then some variable appears twice

15-2/ CFG:

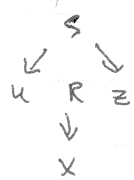
$S \to AX$
$A \to 0$
$X \to YB \mid 1$
$B \to 1$
$Y \to AX$

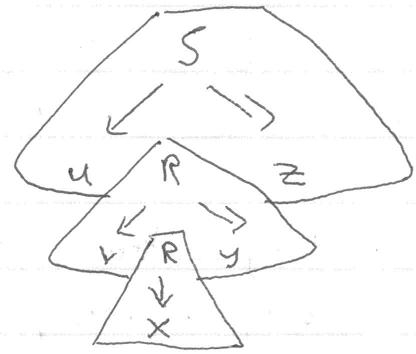
0011. $S \Rightarrow AX \Rightarrow 0X \Rightarrow 0Y1 \Rightarrow 0AX1 \Rightarrow 0011$

$\{0^{n+1}1^{n+1} \mid n \in N\}$

Top

end





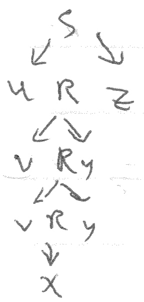
Top

Cycle

end

$u, v, x, y, z \in \Sigma^*$
$S, R \in V$

Top
cyc
cyc
end



PDA → CFG

$V_{start,end} \to u\, V_{p,q}\, z$
$V_{p,q} \to v\, V_{p,q}\, y \mid V_{r,s}$
$V_{r,s} \to x$

$\forall A \in CFL$ $\quad p = 2^{|V|}+1$ (where V is the variables of the smallest CNF CFG)
$\exists p \in N,$
$\forall s \in A,$
$|s| \geq p \to$
$\exists u, v, x, y, z,$
$s = uvxyz$
$\wedge\ (\forall i \in N,\ uv^ixy^iz \in A)$
$\wedge\ |vy| > 0$
$\wedge\ |vxy| \leq p$

Choose A, given p,
Choose s, prove $|s| \geq p$, $s \in A$
Given u, v, x, y, z, s.t. $|vy| > 0$
$|vxy| \leq p$
Choose $i \in N$, prove
$uv^ixy^iz \notin A$

15-3

Ex 1

$A = \{ a^n b^n c^n \mid n \in N \}$

$S = a^p b^p c^p$ (clearly $s \in A$)

$s = uvxyz$ $|vy| > 0$, $|vxy| \leq p$

vxy is either (1) all as, (2) all bs, (3) all cs, (4) abs (5) bc

(1) $vxy = a^j$ $u = a^i$ $z = a^k b^p c^p$ $i + j + k = p$ $j > 0$

$v = a^{j_0}$ $x = a^{j_1}$ $y = a^{j_2}$ $j_0 + j_2 > 0$

choose n, s.t. $uv^n x y^n z \notin A$

$a^i a^{j_0 n} a^{j_1} a^{j_2 n} a^k b^p c^p \in \cancel{A}$ iff

$i + j_0 n + j_1 + j_2 n + k = p = i + j + k = i + j_0 + j_1 + j_2 + k$

$j_0 n + j_2 n = j_0 + j_2$ $n = 1$

(2) + (3)

(4) ≡ (5) $vxy = a^n b^m$ $u = a^l$ $z = b^k c^p$ $n + l = p$ $m + k = p$

v is all as or ~~ab~~ all the as and some bs 4.1, 4.2, 4.3, 4.4

y is all bs or all the bs and some as

4.1 $v = a^n b^d$ $x = b^e$ $y = b^f$ $d + e + f = m$

$uv^i x y^i z = a^l (a^n b^d)^i b^e b^{fi} b^k c^p$

↳ if $i = 0$, then too few as & bs

$i > 1$, then bs come before as

$(a^n b^d)^2 = a^n b^d a^n b^d$

15-4/ $\{a^i b^j c^k \mid 0 \le i \le j \le k\}$

$s = a^p b^{2p} c^{3p}$

$a^p b^{p+1} c^{p+2}$

$\{ww \mid w \in \{0,1\}^*\}$

$s = 0^{p+1} 1 0^{p+1} 1$ $u = 0^p$ $v = 0$ $x = 1$ $y = 0$ $z = 0^p 1$

$v^i x y^i = 0^i 1 0^i$

$u v^i x y^i z = 0^{p+i} 1 0^{p+i} 1$

5 6 7

$s = 0^p 1^p 0^p 1^p$

1 2 3 4

$\{x \# y \mid x, y \in \{0,1\}^*, x \ne y\}$

$0^p \# 0^{2p}$ $uvxy = 0^p$ $z = \# 0^{2p}$

$u = 0^{p-3}$ $v = 000$ $x = \varepsilon$ $y = \varepsilon$

~~$S \to X \# Y$~~

~~$X \to 0X \mid 1X \mid \varepsilon$~~

~~$Y \to 0Y \mid 1Y \mid \varepsilon$~~

$S \to 0\#1 \mid 1\#0 \mid 0\# \mid 1\#$

$\mid ASA$

$A \to 0A \mid 1A \mid \varepsilon$

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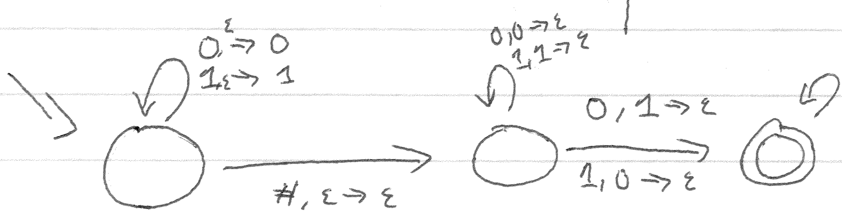
001 # 001



← $\{x \# y^R \mid x \ne y\}$