1-1

① $DFA \Leftrightarrow NFA$ ② REX — Regular Languages REG

(a computer
$f: \Sigma^* \to Y/N$)
set membership

(a programming language:
"declaring the set itself"
$D: REX \to P(\Sigma^*)$)
the set itself

CFL - context-free languages $0^n 1^n \notin REG$

CFG - a context-free grammar $0^n 1^n \in CFL$

$D: CFG \to P(\Sigma^*)$

" 0: $S \to 0S1$ " : CFG
1: $S \to \varepsilon$

" $S \to ASB$ "
$S \to \varepsilon$
$A \to 0$
$\boxed{B \to 1}$

string of variables + terminals

start variable

terminals

a production
or a rule
or a substitution

variables = $\{S, A, B\}$

Σ = alphabet = terminals = $\{0, 1\}$

" S → ◡̈ S ◠̈
S → ε "

(◡̈^{n} ◠̈^{n})
Σ = { ◡̈, ◠̈ }

A rule r is a variable (V)
and a string of variables and terminals
(implicit id) $(V \cup \Sigma)^*$

$D_0 : CFG \Rightarrow \Sigma^*$

$\underline{S} \overset{0}{\Rightarrow} \underline{0\underline{S}1} \overset{0}{\Rightarrow} 00S11 \overset{0}{\Rightarrow}^* 0^{n+2} \underline{S} 1^{n+2} \overset{1}{\Rightarrow} 0^{n+2} \varepsilon 1^{n+2}$
$= 0^{n+2} 1^{n+2}$

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A CFG $g = (V, \Sigma, R, S)$

V = a finite set of variables

Σ = alphabet = a finite set

$(V \cap \Sigma = \emptyset)$

R = an indexed set of $V \times (V \cup \Sigma)^*$

$S \in V$

"$S \to 0S1 \mid \varepsilon$" = "$S \to 0S1$
$S \to \varepsilon$"

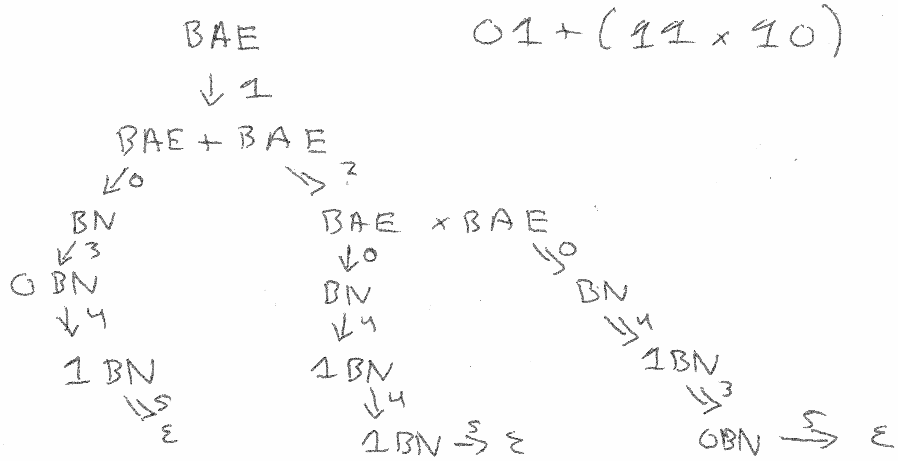
$$BAE = \overset{0}{BN} \mid \overset{1}{BAE + BAE} \mid \overset{2}{BAE \times BAE}$$

binary arithmetic expression

$$BN = \underset{3}{0BN} \mid \underset{4}{1BN} \mid \underset{5}{\varepsilon}$$

$01 + 11 * 10$

$01 + (11 \times 10)$



$BAE \xrightarrow{2} BAE \times BAE \xrightarrow{1} BAE + BAE \times BAE$

~~BAE~~ $(01 + 11) \times 10$

If there is only 1 path (of rules) for every string in the language, then the lang is UNAMBIGUOUS

O.w. ambiguous (No transform exists)

9-3/

$L(g)$ $\qquad\qquad$ $L : CFG \Rightarrow P(\Sigma^*)$

$= \{ w \in \Sigma^* \mid S \Rightarrow^* w \}$

~~A variable V derives string w ($V \Rightarrow^* w$) if~~

A string of var+terminal u derives a string v ($u \Rightarrow^* v$) iff

$$u \Rightarrow^* u \qquad \frac{u \Rightarrow w \quad w \Rightarrow^* v}{u \Rightarrow^* v} \quad w \in (V \cup \Sigma)^*$$

A string u yields v ($u \Rightarrow v$) iff

$uAv \Rightarrow uwv$ where $u \in \Sigma^*$, $v \in (V \cup \Sigma)^*$, $A \in V$, $(A,w) \in R$ $(A \to w)$

$0^n 1^n$: $S \to 0S1 \mid \varepsilon$

"eq no. of 0&1" : $S \to \varepsilon \mid 0S1 \mid 1S0 \mid SS$

palindromes : $S \to \varepsilon \mid 0S0 \mid 1S1 \mid 0 \mid 1$

word repeated $= \{ ww \mid w \in \Sigma^* \}$

valid add $= \{ 0^n 1 0^m 1 0^{n+m} \}$

Closed under union: Given g_1 and g_2, $\exists g_3$

$L(g_1) \cup L(g_2) = L(g_3)$

$S_3 \to S_1 \mid S_2$

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Kleene star : Given G, $\exists H$

$L(H) = L(G)^*$

$A^* = \varepsilon \cup AA^*$

$H.S \rightarrow \varepsilon \mid G.S \; H.S$

(Concat
$S_3 \Rightarrow S_1 S_2$)

What about intersect?
NOT CLOSED

$\exists G_1, G_2 \; \forall G_3, \; L(G_3) \neq L(G_1) \cap L(G_2)$

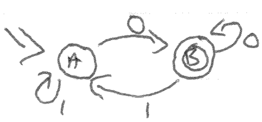
REG $\subseteq$ CFL

$\forall d \in DFA, \exists g \in CFG, \; L(g) = L(d)$

given $d = (Q, \Sigma, q_0, \delta : Q \times \Sigma \Rightarrow Q, F \subseteq Q)$

find $g = (V, \Sigma, R, S)$

ends in 0 :



$S \rightarrow ANY \; 0$

$ANY \rightarrow 0 \, ANY \mid 1 ANY \mid \varepsilon$

$A \rightarrow 0B \mid 1A$

$B \rightarrow \varepsilon \mid 0B \mid 1A$

$V = Q$

$S = q_0$

$\underbrace{q_i \Rightarrow^* w \quad \text{iff} \quad q_i \overset{w}{\Rightarrow^*} q_f}$ $\; q_f \in F$

$(q_i, \varepsilon) \in R$ iff $q_i \in F$

$(q_i, a q_k) \in R$ iff $\delta(q_i, a) = q_k$

Noam Chomsky