

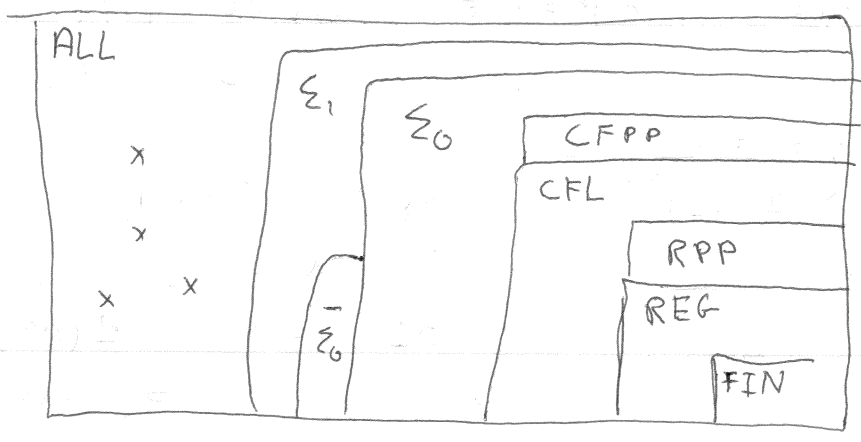
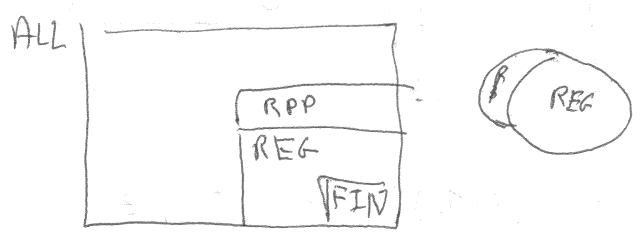
Regular Pumping Property (RPP)

$\forall A \in \text{REG.}$
 $\exists p \in \text{Nat.}$
 $\forall s \in A.$
 $|s| > p \rightarrow$
 $\exists xyz \mid \begin{cases} s = xyz \wedge |y| > 0 \\ \wedge |xy| < p. \end{cases}$
 $\forall i \in \text{Nat.}$
 $xy^i z \in A.$



\neg RPP

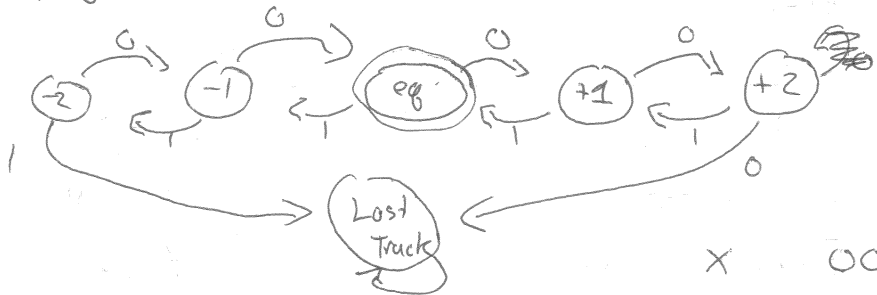
$\exists B \in \text{ALL.}$
 $\forall p \in \text{Nat.}$
 $\exists s \in B.$
 $|s| > p \rightarrow$
 $\forall xyz \mid \begin{cases} s = xyz \wedge |y| > 0 \\ \wedge |xy| < p, \end{cases}$
 $\exists i \in \text{Nat}$
 $\neg (xy^i z \in B)$
 $= xy^i z \notin B$



8-2

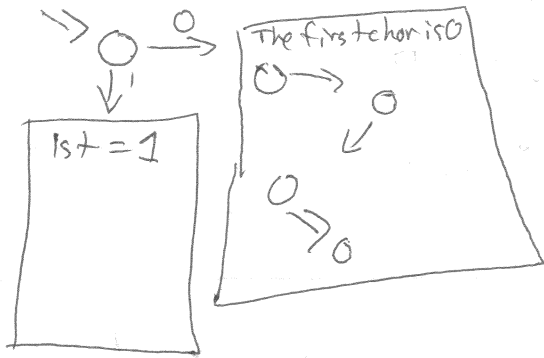
$$C = \{ w \in B^n \mid w \text{ has an equal no. of 0s and 1s} \}$$

C & REG



X 000111
 V 001101010100111020

000111
 ↑
 00x11
 ↑
 0xx1
 ↑
 xxx



$$\forall p \Rightarrow \exists s = (01)^p \Rightarrow \forall xyz$$

$$s = xyz \quad (01)^p = xyz \quad |y| > 0 \quad |xy| < p$$

$$\Rightarrow \exists i =$$

$$xy^iz \in C$$

$$= 0^i 1 (01)^{p-1}$$

$$i + p - 1 = 1 + p - 1$$

$$i \neq 1$$

too many 0s

$$x = 01 \quad y = 0 \quad z = 1 (01)^{p-2}$$

$$x = \epsilon \quad y = 010 \quad z = 1 (01)^{p-1}$$

too many 1s

$$x = \epsilon \quad y = 01 \quad z = (01)^{p-1}$$

$$xy^iz = (01)^i (01)^{p-1} \in C \text{ iff } i + p - 1 = i + p - 1$$

$$0 = 0$$

$$s = 0^p 1^p \Rightarrow \forall xyz$$

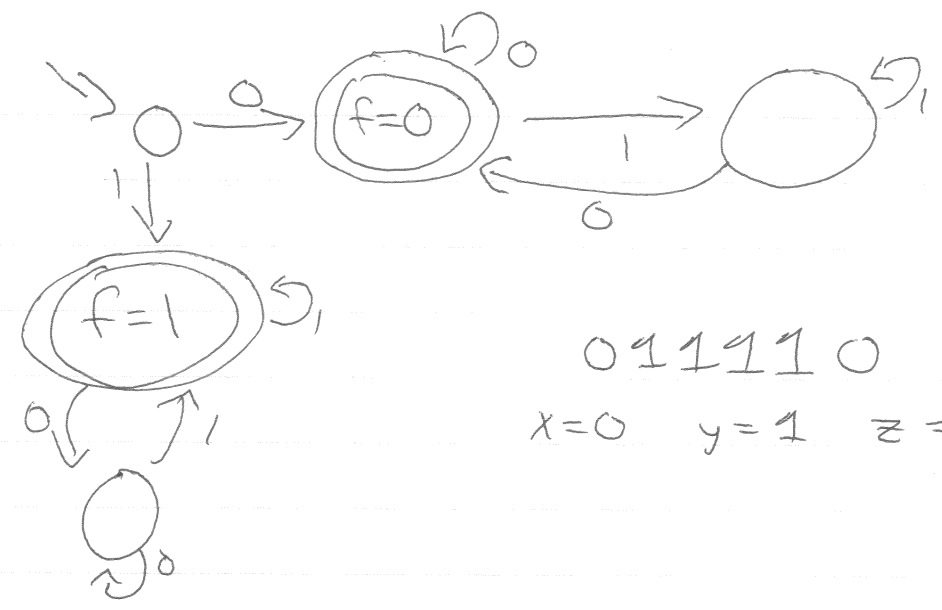
$$x = 0^u \quad y = 0^v \quad z = 0^w 1^p \quad u+v+w = p \quad u+v < p \quad v > 0$$

$$xy^iz = 0^u 0^{vi} 0^w 1^p \quad u+vi+w = p \quad vi = v \quad i = 1$$

Choose 0, 2, 7, etc for i

8-3/

$$D_1 = \{ w \mid w's \text{ first and last char are equal} \}$$



011110
 x=0 y=1 z=1110

$$D_k = \{ w \mid w's \text{ first } k \text{ and last } k \text{ reverse are equal} \}$$

$$E = \{ ww^R \mid w \in B^* \}$$

claim: E is REG

$$s = 0^p x \quad s = 0^p 1^p x \quad s = 0^p 1^p 1^p 0^p$$

$$x = 0^u \quad y = 0^v \quad z = 0^w 1^p 1^p 0^p \quad u+v+w=p$$

$$xy^i z = 0^u 0^v 0^w 1^p 1^p 0^p \quad u+vi+w=p$$

only true if $i=1$

Year	1950	1951	1952	1953	1954	1955	1956	1957	1958	1959	1960	1961	1962	1963	1964	1965	1966	1967	1968	1969	1970	1971	1972	1973	1974	1975	1976	1977	1978	1979	1980	1981	1982	1983	1984	1985	1986	1987	1988	1989	1990	1991	1992	1993	1994	1995	1996	1997	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017	2018	2019	2020	2021	2022	2023	2024	2025	2026	2027	2028	2029	2030	2031	2032	2033	2034	2035	2036	2037	2038	2039	2040	2041	2042	2043	2044	2045	2046	2047	2048	2049	2050	2051	2052	2053	2054	2055	2056	2057	2058	2059	2060	2061	2062	2063	2064	2065	2066	2067	2068	2069	2070	2071	2072	2073	2074	2075	2076	2077	2078	2079	2080	2081	2082	2083	2084	2085	2086	2087	2088	2089	2090	2091	2092	2093	2094	2095	2096	2097	2098	2099	2100																				
Population	150,000,000	155,000,000	160,000,000	165,000,000	170,000,000	175,000,000	180,000,000	185,000,000	190,000,000	195,000,000	200,000,000	205,000,000	210,000,000	215,000,000	220,000,000	225,000,000	230,000,000	235,000,000	240,000,000	245,000,000	250,000,000	255,000,000	260,000,000	265,000,000	270,000,000	275,000,000	280,000,000	285,000,000	290,000,000	295,000,000	300,000,000	305,000,000	310,000,000	315,000,000	320,000,000	325,000,000	330,000,000	335,000,000	340,000,000	345,000,000	350,000,000	355,000,000	360,000,000	365,000,000	370,000,000	375,000,000	380,000,000	385,000,000	390,000,000	395,000,000	400,000,000	405,000,000	410,000,000	415,000,000	420,000,000	425,000,000	430,000,000	435,000,000	440,000,000	445,000,000	450,000,000	455,000,000	460,000,000	465,000,000	470,000,000	475,000,000	480,000,000	485,000,000	490,000,000	495,000,000	500,000,000	505,000,000	510,000,000	515,000,000	520,000,000	525,000,000	530,000,000	535,000,000	540,000,000	545,000,000	550,000,000	555,000,000	560,000,000	565,000,000	570,000,000	575,000,000	580,000,000	585,000,000	590,000,000	595,000,000	600,000,000	605,000,000	610,000,000	615,000,000	620,000,000	625,000,000	630,000,000	635,000,000	640,000,000	645,000,000	650,000,000	655,000,000	660,000,000	665,000,000	670,000,000	675,000,000	680,000,000	685,000,000	690,000,000	695,000,000	700,000,000	705,000,000	710,000,000	715,000,000	720,000,000	725,000,000	730,000,000	735,000,000	740,000,000	745,000,000	750,000,000	755,000,000	760,000,000	765,000,000	770,000,000	775,000,000	780,000,000	785,000,000	790,000,000	795,000,000	800,000,000	805,000,000	810,000,000	815,000,000	820,000,000	825,000,000	830,000,000	835,000,000	840,000,000	845,000,000	850,000,000	855,000,000	860,000,000	865,000,000	870,000,000	875,000,000	880,000,000	885,000,000	890,000,000	895,000,000	900,000,000	905,000,000	910,000,000	915,000,000	920,000,000	925,000,000	930,000,000	935,000,000	940,000,000	945,000,000	950,000,000	955,000,000	960,000,000	965,000,000	970,000,000	975,000,000	980,000,000	985,000,000	990,000,000	995,000,000	1,000,000,000
GDP	100,000,000,000	110,000,000,000	120,000,000,000	130,000,000,000	140,000,000,000	150,000,000,000	160,000,000,000	170,000,000,000	180,000,000,000	190,000,000,000	200,000,000,000	210,000,000,000	220,000,000,000	230,000,000,000	240,000,000,000	250,000,000,000	260,000,000,000	270,000,000,000	280,000,000,000	290,000,000,000	300,000,000,000	310,000,000,000	320,000,000,000	330,000,000,000	340,000,000,000	350,000,000,000	360,000,000,000	370,000,000,000	380,000,000,000	390,000,000,000	400,000,000,000	410,000,000,000	420,000,000,000	430,000,000,000	440,000,000,000	450,000,000,000	460,000,000,000	470,000,000,000	480,000,000,000	490,000,000,000	500,000,000,000	510,000,000,000	520,000,000,000	530,000,000,000	540,000,000,000	550,000,000,000	560,000,000,000	570,000,000,000	580,000,000,000	590,000,000,000	600,000,000,000	610,000,000,000	620,000,000,000	630,000,000,000	640,000,000,000	650,000,000,000	660,000,000,000	670,000,000,000	680,000,000,000	690,000,000,000	700,000,000,000	710,000,000,000	720,000,000,000	730,000,000,000	740,000,000,000	750,000,000,000	760,000,000,000	770,000,000,000	780,000,000,000	790,000,000,000	800,000,000,000	810,000,000,000	820,000,000,000	830,000,000,000	840,000,000,000	850,000,000,000	860,000,000,000	870,000,000,000	880,000,000,000	890,000,000,000	900,000,000,000	910,000,000,000	920,000,000,000	930,000,000,000	940,000,000,000	950,000,000,000	960,000,000,000	970,000,000,000	980,000,000,000	990,000,000,000	1,000,000,000,000																																																																																
Per Capita GDP	666.67	709.68	750.00	787.88	823.53	857.14	888.89	918.18	945.95	972.22	1,000.00	1,027.78	1,055.56	1,083.33	1,111.11	1,138.89	1,166.67	1,194.44	1,222.22	1,250.00	1,277.78	1,305.56	1,333.33	1,361.11	1,388.89	1,416.67	1,444.44	1,472.22	1,500.00	1,527.78	1,555.56	1,583.33	1,611.11	1,638.89	1,666.67	1,694.44	1,722.22	1,750.00	1,777.78	1,805.56	1,833.33	1,861.11	1,888.89	1,916.67	1,944.44	1,972.22	2,000.00	2,027.78	2,055.56	2,083.33	2,111.11	2,138.89	2,166.67	2,194.44	2,222.22	2,250.00	2,277.78	2,305.56	2,333.33	2,361.11	2,388.89	2,416.67	2,444.44	2,472.22	2,500.00	2,527.78	2,555.56	2,583.33	2,611.11	2,638.89	2,666.67	2,694.44	2,722.22	2,750.00	2,777.78	2,805.56	2,833.33	2,861.11	2,888.89	2,916.67	2,944.44	2,972.22	3,000.00	3,027.78	3,055.56	3,083.33	3,111.11	3,138.89	3,166.67	3,194.44	3,222.22	3,250.00	3,277.78	3,305.56	3,333.33	3,361.11	3,388.89	3,416.67	3,444.44	3,472.22	3,500.00	3,527.78	3,555.56	3,583.33	3,611.11	3,638.89	3,666.67	3,694.44	3,722.22	3,750.00	3,777.78	3,805.56	3,833.33	3,861.11	3,888.89	3,916.67	3,944.44	3,972.22	4,000.00																																																				