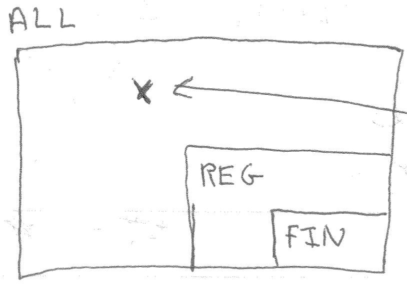
7-1



$REG := DFA = NFA = REX$

$x \in ALL$

$x \notin REG$

$y \in REG$

$\iff$

$\exists d \in DFA.\ L(d) = y$

Prove: $\exists x.\ \underbrace{x \in ALL}_{x \in P(\Sigma^*)}$ but $\underbrace{x \notin REG}_{\neg(x \in REG)}$

$\neg(\exists d \in DFA.\ L(d) = x)$

$\neg(A \wedge B) = \neg A \vee \neg B$

$\neg(\exists x.\ P(x)) = \forall x.\ \neg P(x)$

$\neg(\forall x.\ P(x)) = \exists x.\ \neg P(x)$

$\forall d \in DFA.\ L(d) \neq x$

$P(d) =$ "$L(d) = x$"

$\neg P(d) =$ "$L(d) \neq x$"

$$\exists x \in P(\Sigma^*).\ \forall d \in DFA.\ L(d) \neq x$$

Imagine $F : DFA \to Prop$ and $\forall d \in DFA.\ F(d)$ is true

$F(d) = d$ is a 5-tuple

Suppose that instead $F' : Lang \to Prop$

and $\forall d \in DFA.\ F'(L(d))$

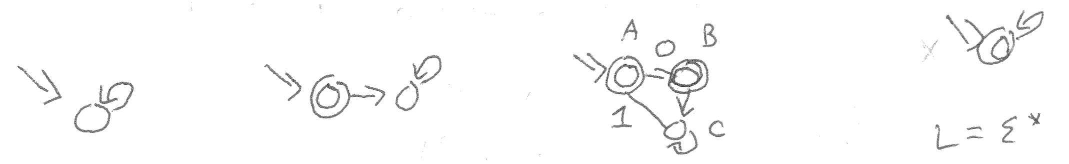
$F'(\ell) =$ "01" $\in \ell$ or "01" $\notin \ell$

Suppose that $\neg F'(x)$

implies that $x \notin DFA$ [really $L(x) \notin DFA$]

7-2

What's the smallest DFA where $|L(d)| = 2$?



$|L| = 0$ $|L| = 1$ $|L| = 2$ $L = \Sigma^*$

$\varepsilon = A$ ✓

$0 = AB$ ✓

$1 = AC$ ✗

$00 = ABC$ ✗

$01 = ABC$ ✗

$11 = ACC$ ✗

Suppose d has many states, and x is $\in L(d)$

how many states could x visit?

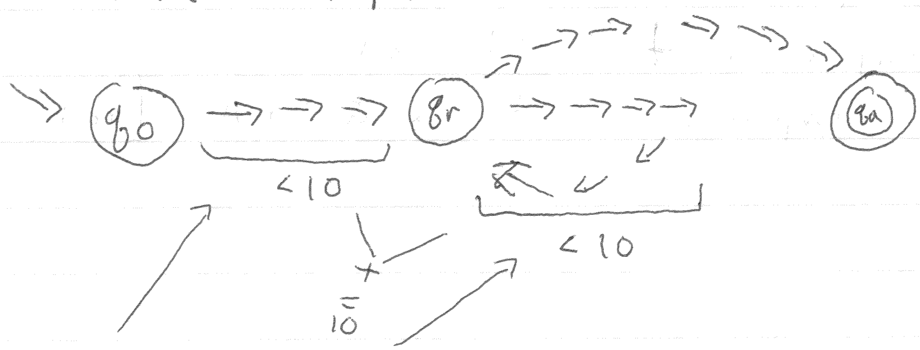
~~$[1, |x|]$~~

$[1, 1 + |x|]$

Suppose d has 10 states and $x \in L(d)$

and $|x| = 15$, what do you know?

Some state is repeated



A B C D E F G H I J E C B D H I

$x =$ before, in between, after

$q_0 \to^* q_r$ $q_r \to^* q_r$ $q_r \to^* q_a$

7-3

$d = |Q|$ states, if $\exists x \in L(d)$, $|x| > |Q|$,

F = then something gets repeated

$x = b\ m\ a \in L(d)$

$b\ a \in L(d)$

$b\ mm\ a \in L(d)$

$\overset{b}{(q_0 \to q_r)}\ \overset{m}{(q_r \to q_r)}\ \underset{a}{(q_r \to q_a)}$

$\to \forall i \in Nat,\ b\ m^i a \in L(d)$

F is about DFAs

want: F' about languages and F' should hold on all DFA langs (i.e. REG)

$F'(A : \text{some language}) =$ Regular Pumping Propperty (RPP)

$\exists p \in Nat.$

$\quad \forall s \in A.$

$\quad\quad |s| \geq p \to$

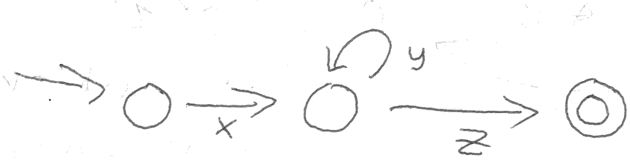
$\quad\quad \exists x y z,\ s = x \circ y \circ z$

$\quad\quad$ where $(\forall i \in Nat.\ x y^i z \in A)$

$\quad\quad\quad |y| > 0$

$\quad\quad\quad |xy| \leq p$

$(\forall d \in DFA.\ F'(L(d)))$



Goal: $\exists B \in ALL.\ \neg F'(B)$

7-4)

$\neg RPP(B) =$

$\forall p \in Nat,$

$\exists s \in B,$

$|s| \geq p \rightarrow$

$\forall xyz. \; s = xyz$

$|y| > 0$

$\wedge \; |xy| \leq p \rightarrow$

$\exists i, \; xy^i z \notin B$

$B = \{ 0^n 1^n \mid n \in Nat \}$

Given : p

Choose : $s = 0^{p+1} 1^{p+1}$

Prove : $|s| \geq p$ $\quad (p+1)+(p+1) = 2p+2 \geq p$ ✓

Given : $x, y, z = s$ $\quad |y| > 0$ $\quad |xy| \leq p$

Choose : i $\quad 0^{p+1} 1^{p+1} = xyz$

$xy = 0^u \quad z = 0^v 1^{p+1}$

$u + v = p+1 \quad u \leq p$

anything but 1

$$\underbrace{x = 0^a \quad y = 0^b \quad a+b = u \quad b > 0}$$

$a+b+v = p+1 \quad a+b \leq p \quad b > 0$

$xy^i z \notin B$ $\quad 0^a 0^{bi} 0^v 1^{p+1} \in B$

iff $\; a+bi+v = p+1$

$a+bi+v = a+b+v$

$i = 1$