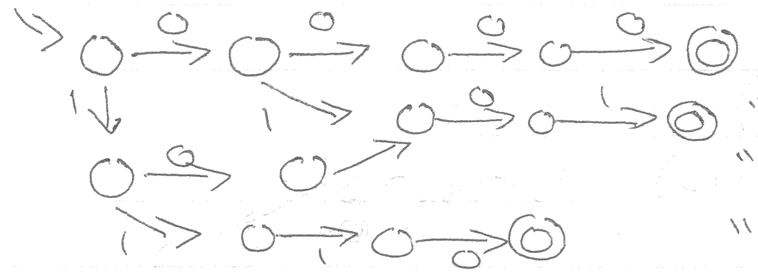
5-1

True add eqns : TAE   "3+3=6" $\in$ TAE
"3+3=7" $\notin$ TAE

1 bit additions



"0+0 =00"
"0+1 =01"
"1+0 =01"
"1+1 =10"

Regular Expression = REX   " wc $\underbrace{*.c}$ " $\to \Sigma^* \circ \bullet \circ c$

- $\emptyset$
- ε
- $c \in \Sigma$
- $r_0 \cup r_1$ (r_0 and $r_1 \in$ REX)
  ($\Sigma = c_0 \cup c_1 \cup \dots \cup c_n$)
- $r_0 \circ r_1$ [Sometimes $r_0 \cap r_1$, r_0^R , r_0^C
- r_0^*

L : spec $\Rightarrow$ set

$L(\emptyset) = \emptyset$ L(mtset) $= \emptyset$

$L(\varepsilon) = \{\varepsilon\}$

$L(c) = \{c\}$

$L(r_0 \cup r_1) = L(r_0) \cup L(r_1)$

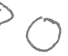
$L(r_0 \circ r_1) = L(r_0) \circ L(r_1)$

$L(r_0^*) = L(r_0)^*$

C : REX $\to$ D~~FA~~ NFA $\longrightarrow$ DFA

6-2

$C(\emptyset) =$



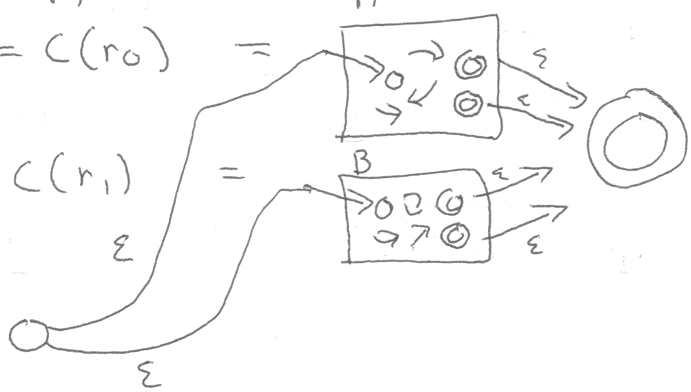
$C(c \in \Sigma) =$

$C(\varepsilon) =$

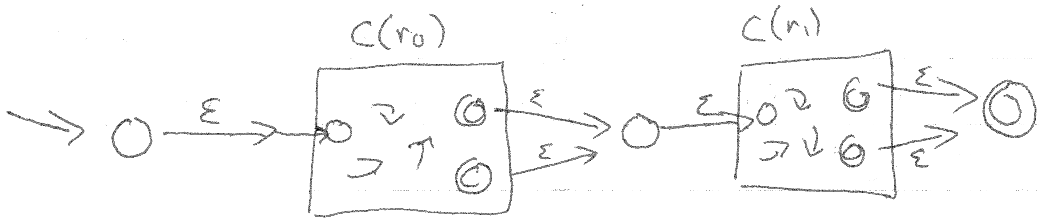
$C(r_0 \cup r_1) =$

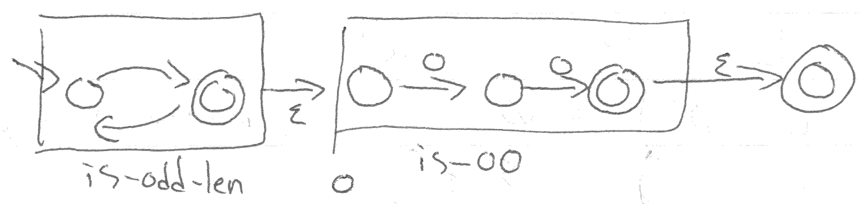
$A = C(r_0) =$

$B = C(r_1) =$



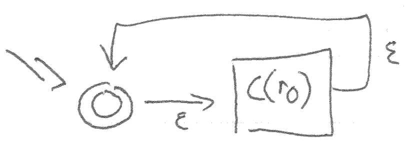
$C(r_0 \circ r_1) =$



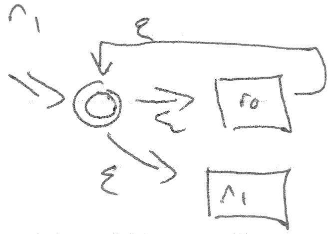


$C(r_0^*) =$

$x^* = \varepsilon \cup x \circ x^*$



$r_0^* \circ r_1$



D : DFA $\rightarrow$ REX

D() $= (\Sigma\Sigma)^* \Sigma$

$D[\cdot] = IN \circ RIP^n \circ OUT$

n-state DFA (i.e. $n = |Q|$)

$IN : n\text{-DFA} \rightarrow (n+2)\text{-GNFA}$

$RIP : (n+1)\text{-GNFA} \rightarrow n\text{-GNFA}$

$OUT : 2\text{-GNFA} \rightarrow REX$

GNFA - a generalized NFA

$= (Q, \Sigma, \underset{\in Q}{q_s}, \Delta, \underset{\in Q}{q_e})$ [NOTICE: 1 final state]

$\Delta : (Q - q_e) \times (Q - q_s) \rightarrow REX$

If $\Delta(q_i, q_u) = r$, then $\forall x \in L(r)$, $q_i \overset{x}{\Rightarrow}{}^* q_u$ in the NFA

$IN = \; d = (Q, \Sigma, q_0, \delta, F)$

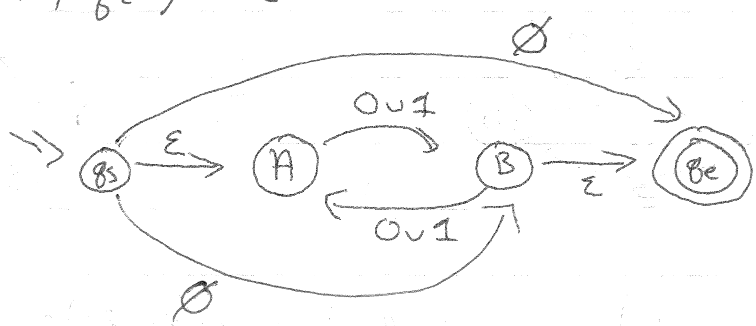
$g = (Q', \Sigma, q'_s, \Delta', q'_e)$

$Q' = Q \cup \{q'_s, q'_e\}$

$\Delta'(q_i, q_u) = c$ s.t. $\delta(q_i, c) = q_u$

$\Delta'(q'_s, q_0) = \varepsilon$

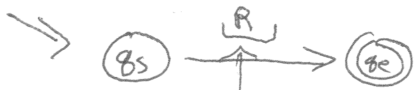
$\Delta'(q_f \in F, q'_e) = \varepsilon$



OUT: 2-GNFA $\rightarrow$ REX

$g = (\{q_s, q_e\}, \Sigma, q_s, \Delta, q_e)$

$\Delta : \underbrace{(Q - q_e)}_{\{q_s\}} \times \underbrace{(Q - q_s)}_{\{q_e\}} \rightarrow REX$



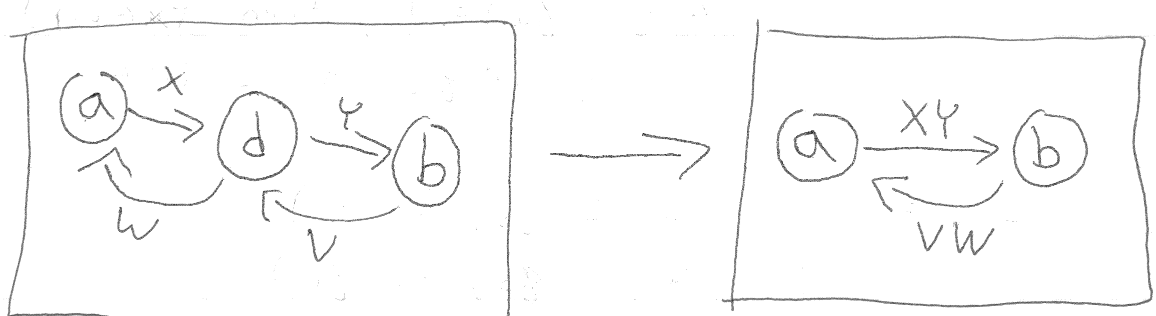
output = R $= \Delta(q_s, q_e)$

RIP: $(n+1)$-GNFA $\rightarrow$ n-GNFA

in $= (Q, \Sigma, q_s, \Delta, q_e)$

output $= (Q', \Sigma, q_s, \Delta', q_e)$

- pick q_d (d = dead) from Q (q_d is NOT q_s or q_e)
  i.e. $Q = Q' \cup \{q_d\}$
- update Δ into Δ', while preserving language

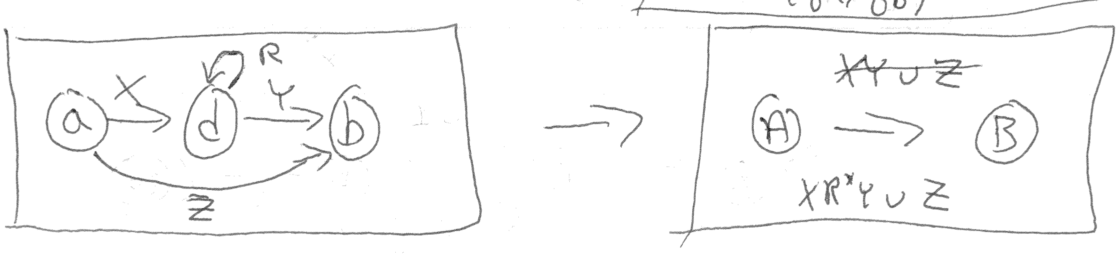


we must figure out XY

forall q_a, q_b

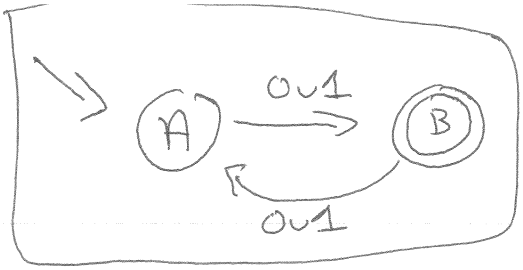
$\Delta'(q_a, q_b) = \Delta(q_a, q_d) \circ \Delta(q_d, q_b) \cup \Delta(q_a, q_b)$

XY , $\Delta(q_a, q_d) = X$, $\Delta(q_d, q_b) = Y$



$\Delta'(q_a, q_b) = \Delta(q_a, q_b) \cup \Delta(q_a, q_d) \circ \Delta(q_d, q_d)^* \circ \Delta(q_d, q_b)$

6-5



$\emptyset \cup x = x$

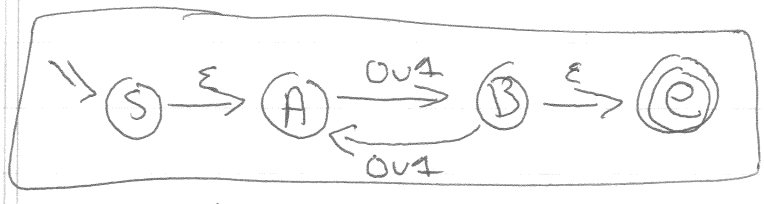
$\emptyset^* = \varepsilon$

$x\emptyset = \emptyset = \emptyset x$

Algebra of Rex

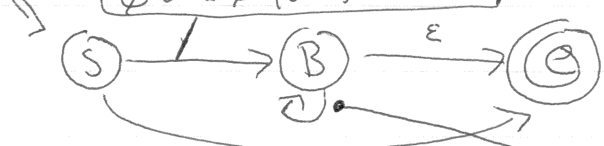
$\varepsilon x = x = x\varepsilon$

$\Downarrow$ IN



$\Downarrow$ RIP(d = a)

$\emptyset \cup \varepsilon\emptyset^*(0\cup 1) = 0\cup 1$,



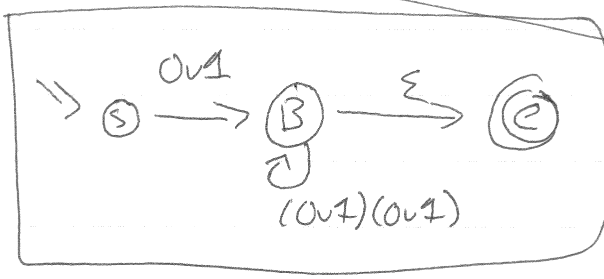
$\emptyset \cup \varepsilon\emptyset^*\emptyset = \emptyset$

$[\emptyset \cup (0\cup 1)\emptyset^*(0\cup 1)]$

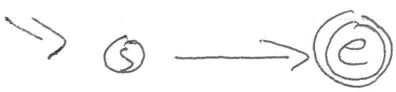
$(0\cup 1)\,\varepsilon\,(0\cup 1)$

$(0\cup 1)(0\cup 1)$

$=$



$\Downarrow$ RIP(d = B)



$\emptyset \cup (0\cup 1)\left((0\cup 1)(0\cup 1)\right)^* \varepsilon$

$=$

$(0\cup 1)\left((0\cup 1)(0\cup 1)\right)^*$

$\Sigma(\Sigma\Sigma)^*$