3-1

What are the possible sets defined by a DFA?

$ALL = P(\Sigma^*)$ = "all possible sets of strings"

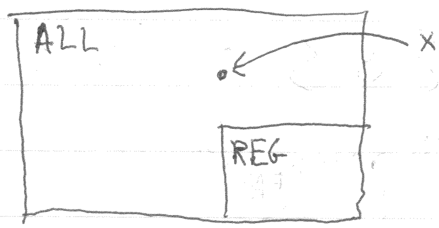
$\emptyset \in ALL$ $\quad$ $\{Jay\} \in ALL$ $\quad$ $Jay \notin ALL$

$\{\epsilon\} \in ALL$ $\quad$ $\epsilon \notin ALL$

Shakespeare Plays $\in$ ALL $\quad$ All valid C programs $\in$ ALL

Solutions to 304 exams $\in$ ALL

x $\in$ ALL, but x $\notin$ REG



ALL $\quad$ DFAs

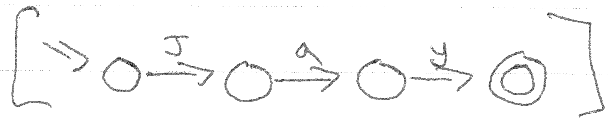
= the regular languages

REG

ALL = REG ✗

ALL $\subseteq$ REG ✗

REG $\subseteq$ ALL ← true



$\in$ DFA $\quad$ L( ) = $\{Jay\}$

$A = \{0^n 1^n \mid n \in Nat\}$ $\quad$ $\epsilon \in A$ $\quad$ $0011 \in A$

$01 \in A$

Why are Q and Σ finite in the DFA defn?

- rigor of math (infinite doesn't exist until after DFAs do)
- reflects reality

ALL $\quad$ REG $\quad$ FIN = finite

= set of all finite sets

REG $\subseteq$ FIN ✗ $\quad$ evens

REG = FIN ✗

FIN $\subseteq$ REG ✓ — there's a DFA for every finite set

digital trie $\quad$ Disjoint

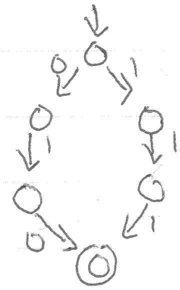
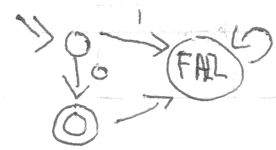
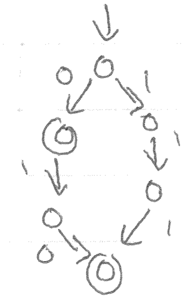
3-2

DFAs run in $\Theta(n)$ time
and $\Theta(\log_2 |Q|)$ space

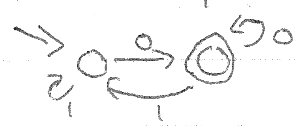
DFAs describe languages
and languages have "operations"
$(\cup, \cap, {}^R, \circ, {}^*, {}^n)$

$\Rightarrow$ do DFAs have "operations"?

$\{010, 111\} \cup \{0\} = \{0, 010, 111\}$

evens (evenlength) $\cup$ eevens (binary number is even)

= string is even long or ends in 0

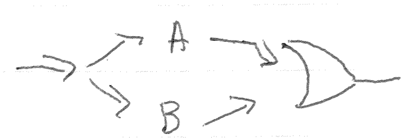
Closure under Union of REG :=

$\forall A, B \in REG.\ A \cup B \in REG.$

$\equiv \forall A, B \in DFA.\ \exists C \in DFA.\ L(C) = L(A) \cup L(B)$

forall then exists $\rightarrow$ a compiler

Input: $A = (Q_A, \Sigma, q_{0A}, \delta_A, F_A)$
$B = (Q_B, \Sigma, q_{0B}, \delta_B, F_B)$



Output: $C = (Q_C, \Sigma, q_{0C}, \delta_C, F_C)$

$Q_C = Q_A \times Q_B$

$q_{0C} = (q_{0A}, q_{0B})$

$\delta_C =$

$\delta_C((q_a, q_b), c) = (\delta_A(q_a, c), \delta_B(q_b, c))$

$F_C = F_A \cancel{\times} F_B$ $\quad \overbrace{Q_A \times F_B}^{B \text{ says } Y} \cup \overbrace{F_A \times Q_B}^{A \text{ says } Y}$ / F_C for $\cap = F_A \times F_B$

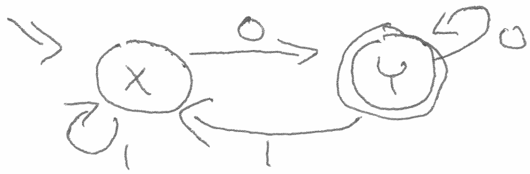
3-3

even length =

00 01

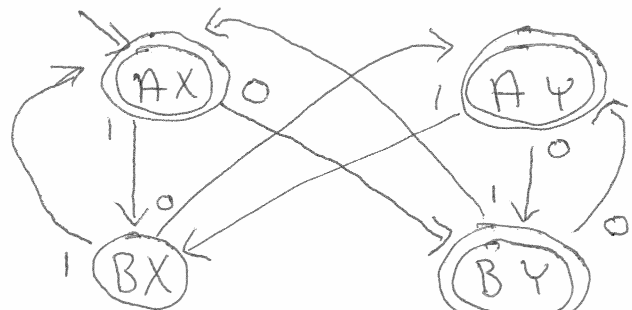
10 11

is-even =



$\cap$ is same but AY stands alone

$\cup$ =



0011 ✓

110 ✓

111

1110

How big is the $\cup$ output?

Space matters $\log(|Q_C|)$

$\log(|Q_A| \cdot |Q_B|)$

$\log(|Q_A|^2)$

$2 \cdot \log(|Q_A|)$

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operations = $\underbrace{\cup, \cap,}_{\text{proved}}$ $\circ$, ^{R}, $*$, ^{n}

"The regular operations"

$\varepsilon \circ X = X$ $\qquad (A \cup X) \cup (Y \cup A) =$

$A \cup (X \cup Y)$