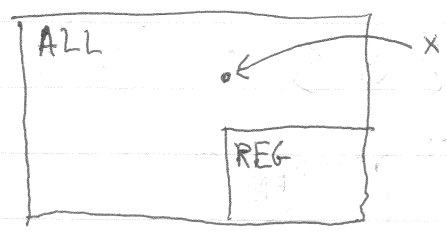


What are the possible sets defined by a DFA?

ALL =  $P(\Sigma^*)$  = "all possible sets of strings"

$\emptyset \in ALL$        $\{Jay\} \in ALL$        $Jay \notin ALL$   
 $\{\epsilon\} \in ALL$        $\epsilon \notin ALL$

Shakespeare Plays  $\in ALL$       All valid programs  $\in ALL$   
 Solutions to 304 exams  $\in ALL$



$x \in ALL$ , but  $x \notin REG$

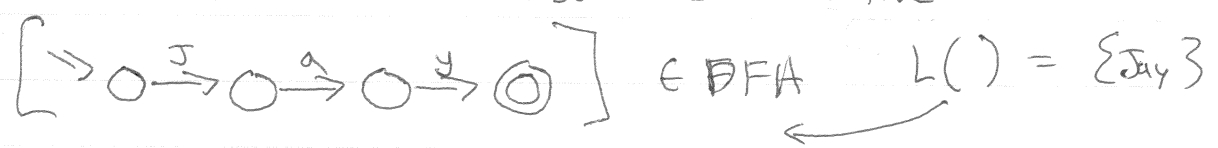
ALL

DFA's = the regular languages  
 REG

ALL = REG X

ALL  $\subseteq$  REG X

REG  $\subseteq$  ALL  $\leftarrow$  true



$\in$  DFA       $L() = \{Jay\}$

$A = \{0^n 1^n \mid n \in \text{Nat}\}$        $\epsilon \in A$        $0011 \in A$   
 $01 \in A$

Why are Q and  $\Sigma$  finite in the DFA defn?

$\hookrightarrow$  rigor of math (infinite doesn't exist until after DFA's do)

$\hookrightarrow$  reflects reality

ALL

REG

FIN = finite

= set of all finite sets

REG  $\subseteq$  FIN X evens

REG = FIN X  $\curvearrowright$

FIN  $\subseteq$  REG  $\leftarrow$  there's a DFA for every

finite set

$\checkmark$   
 digital trie

Disjoint



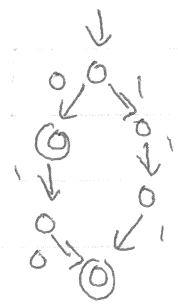
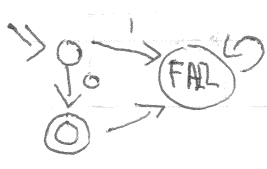
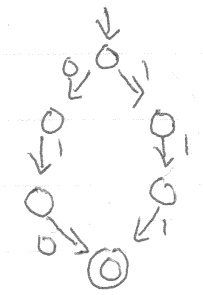
3-2/

DFA's run in  $\Theta(n)$  time and  $\Theta(\log_2 |Q|)$  space

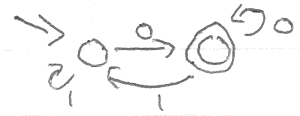
DFA's describe languages and languages have "operations"  $(\cup, \cap, R, \cdot, *, \dots)$

→ do DFA's have "operations"?

$$\{010, 111\} \cup \{0\} = \{0, 010, 111\}$$



evens (even length)  $\cup$  eevens (binary number is even)



= string is even long or ends in 0

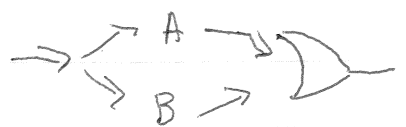
Closure under Union of REG :=

$$\forall A, B \in REG. A \cup B \in REG.$$

$$\equiv \forall A, B \in DFA. \exists C \in DFA. L(C) = L(A) \cup L(B)$$

for all then exists → a compiler

Input:  $A = (Q_A, \Sigma, q_{0A}, \delta_A, F_A)$   
 $B = (Q_B, \Sigma, q_{0B}, \delta_B, F_B)$



Output:  $C = (Q_C, \Sigma, q_{0C}, \delta_C, F_C)$

$$Q_C = Q_A \times Q_B$$

$$\delta_C((q_a, q_b), c)$$

$$q_{0C} = (q_{0A}, q_{0B})$$

$$= (\delta_A(q_a, c), \delta_B(q_b, c))$$

$$\delta_C =$$

$$F_C = F_A \cup F_B$$

$$\frac{B \text{ says } Y}{Q_A \times F_B} \cup \frac{A \text{ says } Y}{F_A \times Q_B}$$

$$F_C \text{ for } \cap = F_A \times F_B$$

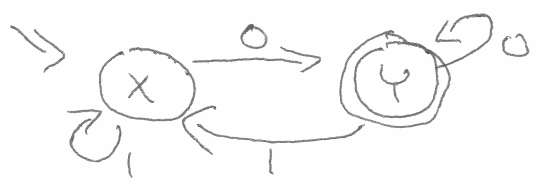
3-3/

even length =



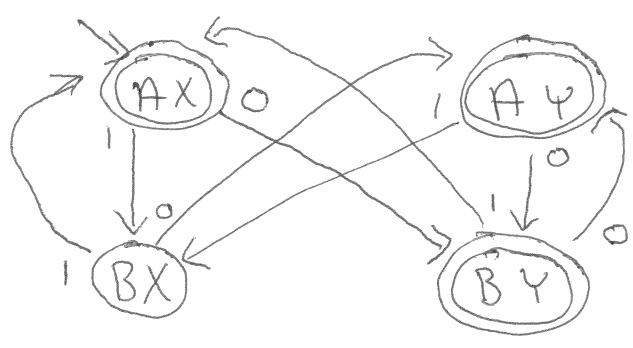
00 01  
10 11

is-even =



n is same but AY stands alone

U =



0011 ✓  
110 ✓  
111  
1110

How big is the U output?

Space matters

$$\log(|Q_c|)$$

$$\log(|Q_A| \cdot |Q_B|)$$

$$\log(|Q_A|^2)$$

$$2 \cdot \log(|Q_A|)$$

operations =  $\cup, \cap, \circ, R, \star, \dots$   
 proved

"The regular operations"

$$\epsilon \circ X = X$$

$$(A \cup X) \cap (Y \cup A) = A \cup (X \cap Y)$$

Handwritten notes at the top of the page, including a circled word and some illegible scribbles.

Handwritten notes in the second section, featuring a large, faint scribble that appears to be a signature or a large word.

Handwritten notes in the third section, containing several lines of text and a prominent diagonal scribble.

Handwritten notes in the fourth section, including a large, dark scribble that obscures some of the text.

Handwritten notes in the fifth section, appearing as several lines of text with some faint markings.

Handwritten notes in the sixth section, including a large, dark scribble and some illegible text.