4-1

$A_{CFG} = \{ \langle G, w \rangle \mid G \in CFG, w \in \Sigma^*, \text{ and } G \text{ derives } w \}$

$\langle S \to 0S1 \mid \varepsilon, 0011 \rangle \in A_{CFG}$

1 Starting from S
2 Non-deterministically apply rules
3 if you generate w, then accept

Tape 1: G
Tape 2: w = 0011
Tape 3: S → 0S1 → □0S11 → _ _ 11

definitely in Σ_1 (accepter)
not in Σ_0 (decider) because it keeps trying

idea 1: count terminals in tape 3, if $> |w|$, reject (stop on this path)
breaks on: $S \to SS$

idea 2: assume / translate G into Chomsky Normal Form
$S \to \varepsilon$ $A \to BC$ ($B, C \neq S$) $A \to t$
there's never more than $2^{|w|}$ rule applications to generate $|w|$
$\in \Sigma_0$

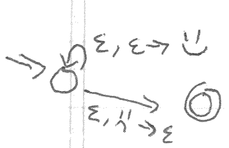
REG ⊂ Deterministic CFL ⊂ CFL

$P \subset \Sigma_0$ $EXP \subset \Sigma_0$
$NP \subset \Sigma_0$

REG ⊂ Σ_0 (compiler)

(CFL ⊂ Σ_0) ⟺
($\forall G \in CFG. \exists T \in \Sigma_0$.)
Pf. T = On input w,
run A_{CFG} ⟨G, w⟩

E_{CFG} = is a CFG's language empty?
idea: check which vars go to terminals, back-propagate this info, then check S
(n^2 in number of rules)



(vars going terminals, Other vars) = $(\emptyset, V)$

24-2)

$A_{TM} = \{ \langle T, w \rangle \mid T \in TM$ and $w \in \Sigma^*$ and T accepts $w \}$

A_{TM} is solved by U

U = " On input $\langle T, w \rangle$, simulate T on w "   Interpreter

$A_{TM} \in \Sigma_1$ ✓

$A_{TM} \in \Sigma_0$? X $\langle T, w \rangle$ where $w \in L(T)$ it rejects

U doesn't decider

PROMISE: Prove that it's not

(Σ_0) SQL, Datalog, ld linker-script, Linux/Cisco firewall rules, regexps (but not Perl), Coq, Agda, ML module linker

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Georg Cantor

N (naturals $= 0, 1, \ldots, 42, \ldots, 1978 \ldots$)
are infinite

E (evens $= 0, 2, 4,$ etc)
are infinite

Z (intergers $= \ldots, -1, 0, 1, \ldots$)
infinite

Q (rationals $= 0/1, 0/2, -1/3, 1/4, \ldots$)
infinite

R (reals $= 0, 1.2, \pi, e, \sqrt{2}, \ldots$)

R_∞ (extended reals) $= R \cup \{-\infty, +\infty\}$

Two sets are the same size if we can turn elements of one into the other and back and not lose information.

($|X| = |Y|$)
(samesize (X, Y))

$\exists$ a mapping function, f with 2 props:

one-to-one = $\forall a, a' \in A$, $b \in B$,
$f(a) = b \Rightarrow$
$f(a') = b \Rightarrow$
$a = a'$.

onto = $\forall b \in B$, $\exists a \in A$. $f(a) = b$

N is the same size as E
$f(n) = 2 \cdot n$

1-to-1: Given a, a', and b
$2 \cdot a = b$
$2 \cdot a' = b$
$\Rightarrow a = a'$ ✓ ✓

onto: $\forall b, \exists a.\ f(a) = b$
$b = 2 \cdot n$
find a, s.t. $2 \cdot a = 2 \cdot n$
$a = n$

A set X is countable if X is the same size as N
$|X| = \aleph_0$

Next... Z? Q? R?

Foreshadowing ~ ~ ~

Is TM the same size as ALL?