Turing Machine Closure Properties

useful = Σ_0 = languages that have deciders = Always say Y/N

theoretic = Σ_1 = languages that have accepters = Sometimes diverge

Union : A set X is closed under union if

$\forall A \in X, \forall B \in X, A \cup B \in X$

Σ_0: Two TMs : A and B

Goal : TM C s.t. $L(C) = L(A) \cup L(B)$

If $A(w) = Y$, then $C(w) = Y$

and if $B(w) = Y$, then $C(w) = Y$

```
bool C (str w) {
    ret A(w) || B(w);
}
```

✓

Σ_1: Because of looping, can't choose to go first

✓ Strat1: 2-tape machine $Q_c = Q_a \times Q_b \cup \{halt\}$

Strat2: ND machine w/ 1 nondet choice: run A or run B

$A_a \times Q_b \to q_a$

$Q_a \times B_a \to q_a$

Intersect : ✓ Σ_0 = ~~DD~~

✓ Σ_1 = run at same time → $A_a \times B_a \to q_a$ but do ~~DD~~

M = if A(w) then B(w) o.w reject

Complement : Given $A \in TM$, find B s.t. $L(B) = L(A)^C$

✓ Σ_0 : B(w) = if A(w) then R o.w. A

✗ Σ_1 : B(w) = reject if A(w) accepts

accept if A(w) rejects

new in Σ_1 → accept if A(w) diverges ←

concat : $A \in TM$, $B \in TM$

find $C \in TM$, $L(C) = L(A) \circ L(B)$

$A = \{ w \# w \}$ $B = \{ 0^n + 0^m = 0^{n+m} \}$

$C = \{ w \# w \; 0^n + 0^m = 0^{n+m} \}$

C's job : find the dividing line

$C(x)$ = for each dividing line (ie for each a, b s.t. $ab = x$) Suppose $|x| = 3$ $|x|+1$

$A(a)$ && $B(b)$

Σ_1 & $C(x)$ = select a dividing line nondeterministically Σ_0 ✓

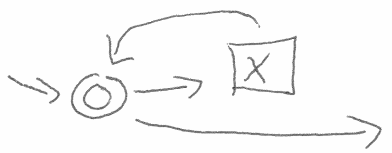
Kleene star : $A \in TM$, find $C \in TM$, $L(C) = L(A)^*$

$X^* = \varepsilon \cup X \circ X^*$ $w \in X^*$ iff

$\exists n.\ w = w_0 \circ \dots \circ w_n$

and $w_i \in X$

and $w_i \neq \varepsilon$



$C(x)$ = non-det choose $n \in [0, |x|]$

non-det divide x into n -substrings

for $i \in [0, n]$, $A(x_i)$ if Reject, reject

~~Rej~~ accept

Σ_0 ✓

Σ_1 ✓

Functional composition (transducers)

$A \in TM$ $B \in TM$ find

given $C \in TM$ s.t.

$C(w) = B(A(w))$

Σ_0 ✓

Σ_1 ✓

12-3

Church-Turing (Hypo-) Thesis

Alonzo ↙ λ > Alan ↓ TM