2-1

{ eggs, toast, jam, milk, butter }

{ eggs, jam, toast, milk, butter }

MB = { e, j, t, m, b }

APB = "All possible breakfasts"

$\forall \Sigma,$

$\mathcal{U}$ = the universe of elements

in this class, $\mathcal{U}$ = "strings"

A string is a sequence of characters

→ Σ^* ; sequence → finite, infinite ; characters → 8 bits in C, some finite set = Σ (sigma) = the alphabet

finite = Nat x Inf⌒ = (Nat ⇒ elem)

X^* (20, (f(x) = 'j'))

j ... j

~~(dx, x)~~

empty = ε (epsilon) = (0, f(x) = ...) ≟ ''

reverse = s^R xyz^R = zyx

$a^3 = aaa$

concat = $s \circ t$, st ($s, t \in$ Strings)

s = abc t = xyz $s \circ t = st$ = abcxyz

$\forall i.\ s^i \in s^*$

exponentiation = Kleene Star = s^* $s^* = \varepsilon \cup s \circ s^*$

: string → set(string) = Language $= \varepsilon \cup s \cup ss \cup sss \cup ssss...$

$(abc)^* = A$ $\boxed{\varepsilon \in A}$ abcabcabcabc $\in A$

abc $\in A$

{ab, cd} $\circ$ {xy, yz} = { abxy, abyz, cdxy, cdyz }

Lexicographic Ordering of Σ^*

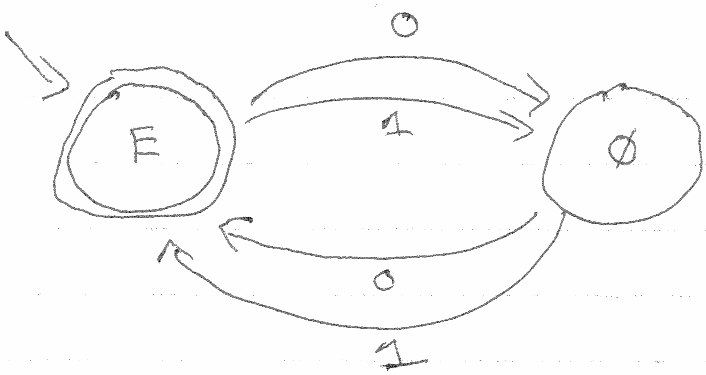
$\Sigma = \{0,1\}$ = a sequence

ε, $\underbrace{0, 1}$, $\underbrace{00, 01, 10, 11}$

$$\sum \quad \boxed{\varepsilon \in A}$$

epsilon is a member of

2-2 "Binary strings of even length" = EvenB



Deterministic Finite Automata = DFA

0100   111   110

○ is a state

→○ is THE start state

$a \xrightarrow{t} b$ is a transition = "If you are in state a and see a 't', then go to b"

◎ is an accept state = "If you stop here, say "Yea!" "

$\varepsilon \Rightarrow$ Yea

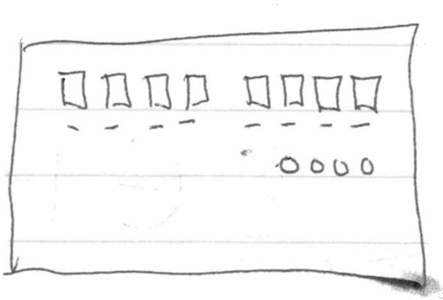
$|1| \Rightarrow$ go to Ø, say no $|s| = 2 \times n$

$|1| + |1| \Rightarrow$ E, yea

Sets A equiv $\longleftrightarrow$ their membership function $x \in A$

the computation

$a \xrightarrow{c,d} b \equiv a \xrightarrow{c} b$, $a \xrightarrow{d} b$ $a \rightarrow b \equiv a \xrightarrow{\varepsilon} b$



2-3

Machine with outputs

Input alphabet $= \Sigma$ Output alphabet $= \Gamma$ (gamma)

Mealy $\quad (a) \xrightarrow[u]{t} (b) \quad t \in \Sigma, \; u \in \Gamma$ "If you read t, then write u"

Moore $\quad (a/u) \xrightarrow{t} (b/v) \quad t \in \Sigma, \; u, v \in \Gamma$ "If you are in state b, print v"

UML Statechart

A DFA is a 5-tuple $=$

$$(\Sigma, Q, q_0, \underset{\text{delta}}{\delta}, F)$$

Σ = an alphabet - finite set

Q = the states = a finite set

q_0 = the start state $\in Q$

δ = the transition function $t \quad \underset{\text{from}}{Q} \times \underset{\text{input}}{\Sigma} \to \underset{\text{to}}{Q}$ ~~output~~

F = the accepting states $\subseteq Q$

Suppose $\Sigma = \{0,1\}$, $|Q| = 4$, how many DFAs are there?

$$2^2 \; \underset{q_0}{4} \times \underset{F}{2^4} \times 4^8 \quad 2^{2 \cdot 8} \quad 2^{16}$$

$$= 2^{22} = 4 \text{ mb}$$

L : Language of the DFA

DFA $\Rightarrow$ Σ^*

$L(d) = \{ x \in \Sigma^* \mid$ ~~...~~ x is accepted by $d \}$

A string x is accepted by DFA d iff

$q_0 \xRightarrow{x}{}^{*} q_i$ such ~~that~~ (st) $q_i \in F$

A DFA d runs from q_i to q_j via x ($q_i \xRightarrow{x}{}^{*} q_j$) iff

$q_i \xRightarrow{\varepsilon}{}^{*} q_i$ $\quad$ $q_i \xRightarrow{ax}{}^{*} q_k$ iff $q_i \xRightarrow{a} q_j$, $q_j \xRightarrow{x}{}^{*} q_k$ $\quad$ $q_{i,j,k} \in Q$, $a \in \Sigma$, $x \in \Sigma^*$

A DFA d steps from q_i to q_j on a ($q_i \xRightarrow{a} q_j$) iff

$((q_i, a), q_j) \in \delta$

$\simeq \quad \delta(q_i, a) = q_j$