6-1

$\neg$CFPP

$\exists A \in ALL.$

$\forall p \in N$

$\exists (s \in A \mid |s| \geq p)$

$\forall (u,x,v,y,z \in \Sigma^* \mid s = uvxyz \wedge |vxy| \leq p \wedge |vy| > 0)$

$\exists i \in N.$

$uv^i xy^i z \notin A$

~~$S \Rightarrow A$~~ ~~B~~

$A \Rightarrow$ ~~ε~~

$S \Rightarrow XC \mid AY$

$C \Rightarrow \varepsilon \mid cC$

$A \Rightarrow \varepsilon \mid aA$

$X \Rightarrow \varepsilon \mid aXb$

$Y \Rightarrow \varepsilon \mid bYc$

$A = \{ a^n b^n c^n \mid n \geq 0 \}$

Given p

$s = a^p b^p c^p \quad |s| = 3p \geq p$

Given $u, v, x, y, z \quad s = uvxyz = a^{\hat{u}} b^{\hat{v}} c^{\hat{p}}$

$|vxy| \leq p \Rightarrow$ ① ~~alls~~ only a ② ~~alls~~ only b ③ ~~alls~~ only c

④ contain a & b ⑤ contain b & c

⑥ ~~a, b, c~~ $\geq p+2 \leq p \Rightarrow$ ~~not~~ possible

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①,②,③ Assume $vxy = a^{\ell} \quad \ell \leq p$

$u = a^{\vec{u}} \quad v = a^{\vec{v}} \quad x = a^{\vec{x}} \quad y = a^{\vec{y}}$

$s = uvxyz = a^{\hat{u}} a^{\hat{v}} a^{\hat{x}} a^{\hat{y}} a^{\hat{z}} b^p c^p$

$a^{\ell} = a^{\vec{v}} a^{\vec{x}} a^{\vec{y}} \quad \ell = \vec{v} + \vec{x} + \vec{y}$

$z = a^{\vec{z}} b^p c^p$

$\hat{u} + \hat{v} + \hat{x} + \hat{y} + \hat{z} = p$

$\hat{v} + \hat{y} > 0$

$\hat{v} + \hat{x} + \hat{y} \leq p$

$uv^i xy^i z \in A$

$\Leftrightarrow \hat{u} + \vec{v}\, i + \hat{x} + \vec{y}\, i + \hat{z} = p$

$\Leftrightarrow (i-1)\hat{v} + (i-1)\hat{y} = 0$

$\Leftrightarrow (i-1)(\vec{v} + \vec{y}) = 0$

$\Leftrightarrow i = 1$

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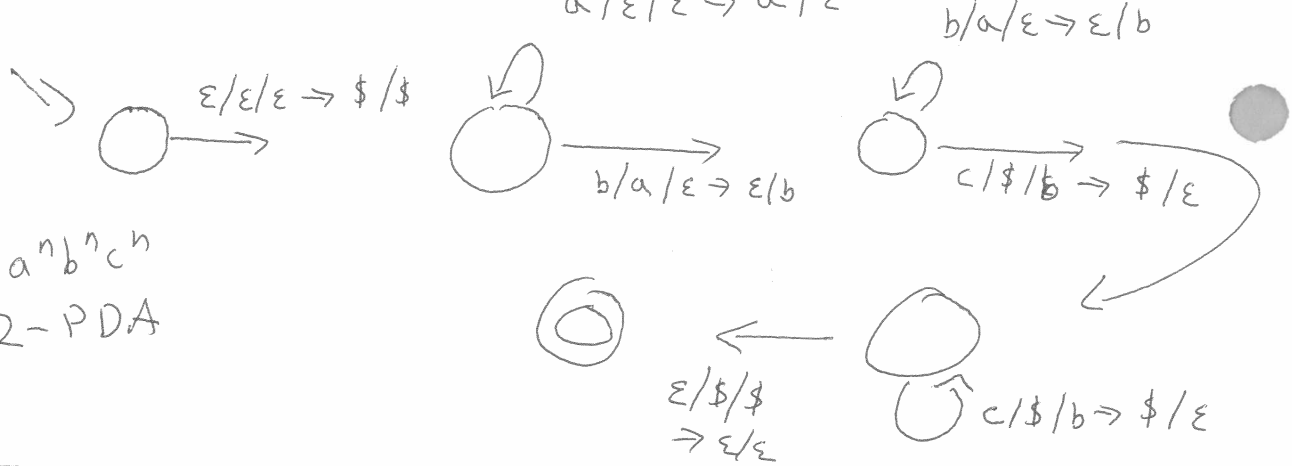
④,⑤ Assume $vxy = a^{\ell} b^m$ (assume split in x)

$s = \underbrace{a^{\hat{u}}}_{u} \underbrace{a^{\hat{v}}}_{v} \underbrace{a^{\hat{x}_1} b^{\hat{x}_2}}_{x} \underbrace{b^{\hat{y}}}_{y} \underbrace{b^{\hat{z}} c^p}_{z}$

$\hat{u} + \hat{v} + \hat{x}_1 = p \quad \hat{v} + \hat{y} > 0$

$\hat{x}_2 + \hat{y} + \hat{z} = p \quad \hat{v} + \hat{x}_1 + \hat{x}_2 + \hat{y} \leq p$

$uv^i xy^i z \in A \Leftrightarrow \underbrace{\hat{u} + \vec{v}\, i + \vec{x}_1}_{\#a} = \underbrace{\hat{x}_2 + \vec{y}\, i + \vec{z}}_{\#b} = \underbrace{p}_{\#c} \Leftrightarrow i = 1$



$a^n b^n c^n$
2-PDA

$(a^n b^n c^n)^i (a^m b^m c^m)^i$ (3-PDA-able)

$\{ a^i b^j c^k \mid 0 \le i \le j \le k \}$

$s = a^p b^{2p} c^{3p}$ ✗ $u = a^p b^{2p} c^{3p-1}$ $v = c$ $x = y = z = \varepsilon$

$a^p b^{p+1} c^{p+2}$ ✗

$a^p b^p c^p \Rightarrow a^p b^p c^{p-1}$ ✓

$\{ a^i b^j c^k \mid 0 < i < j < k \}$

$s = a^p b^{p+1} c^{p+2}$

$\{ w w \mid w \in \Sigma^* \}$ $(\text{palin} = \{ w w^R \mid w \in \Sigma^* \})$

$s = 0^{p+1} 1 0^{p+1} 1$ $u = 0^p$ $v = 0$ $x = 1$ $y = 0$ $z = 0^p 1$

$s = 0^p 1^p 0^p 1^p$

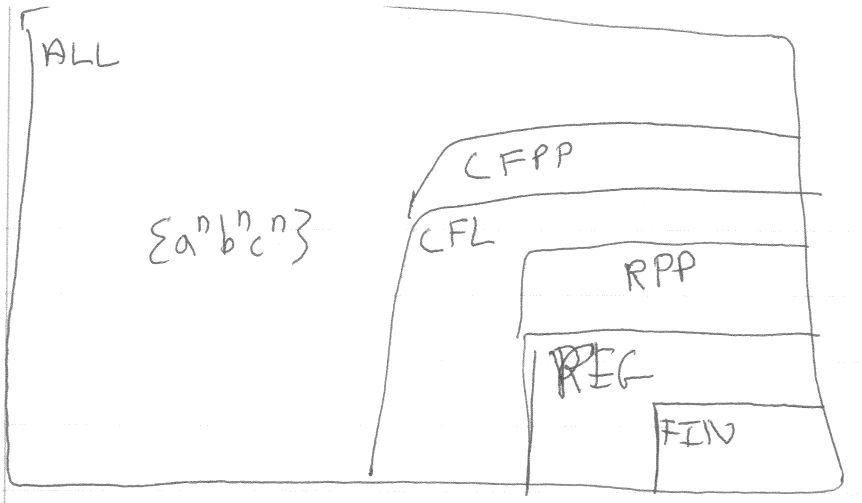
$\{ x \# y \mid x, y \in \Sigma^*, x \neq y \}$

$s = 0^p \# 0^p 1$

$s = 0^p 1^p \# 0^p 1^{p+1}$

$\{ w \# x \mid w, x \in \Sigma^x, w = uxv \text{ (ie } x \text{ is a substring)} \}$

$s = 0^p 1^p \# 0^p 1^p$



DFA ⇔ NFA ⇔ REX

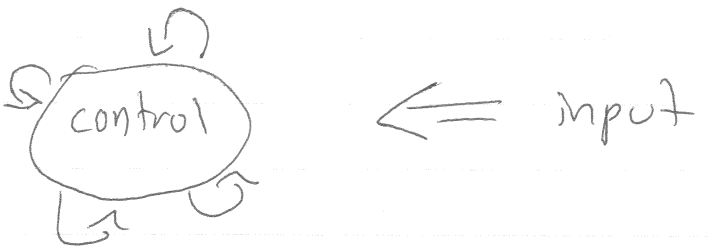
↓

CFG ⇔ PDA

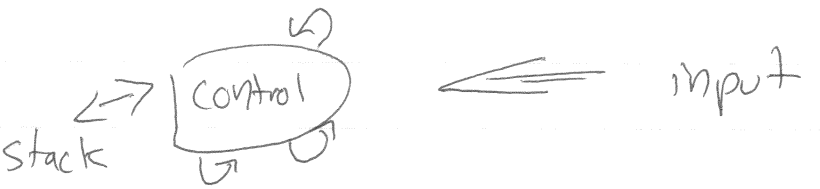
↓

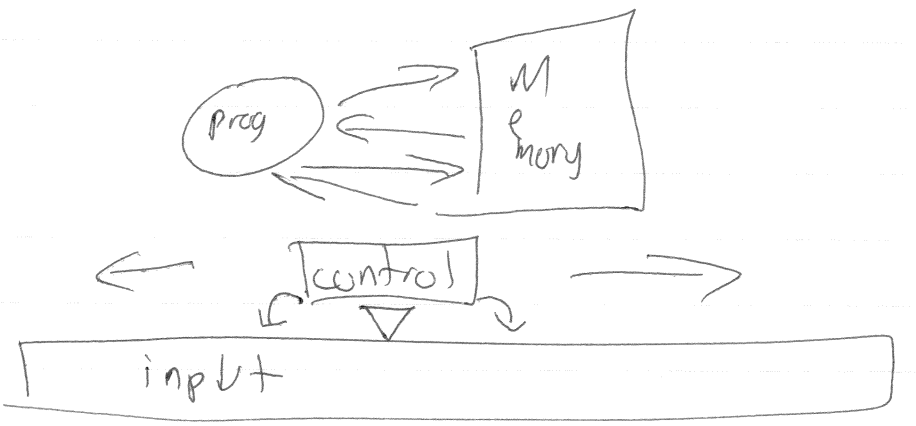
new = Turing Machine
TM

DFA



PDA





$a^n b^n c^n$

a a b b c c
à a b b c c
à a b̀ b c c
à a b̀ b ć c
à à b̄ b ć c
à à b̀ b̀ ć c
à à b̀ b̀ ć ĉ

Ok!