

ALL = CFL?

$y \in ALL$ but $y \notin CFL?$

$x = 0^n 1^n$

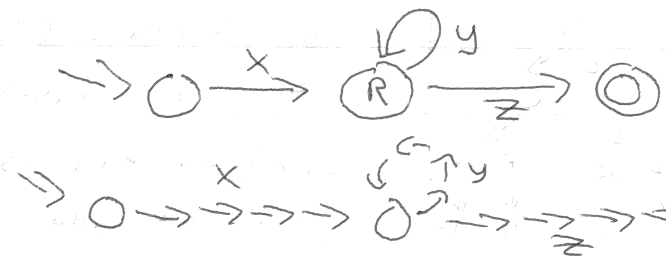
P where $\forall A \in REG, P(A)$

then we proved $\neg P(x) \Rightarrow x \notin REG$

new P' where $\forall A \in CFL, P'(A)$

then we will prove $\neg P'(y) \Rightarrow y \notin CFL$

RPP says if an accept string is long enough then $S = xyz$



"long enough" = $|s| > |Q| \Rightarrow$ a state is repeated (ie R exists)

P' will be based on CFG rather than PDA

use Chomsky Normal Form

$S \rightarrow \epsilon$ (maybe)

$A \rightarrow +$

$A \rightarrow BC$ ($B \& C \neq S$)

$\forall A \in CFL, P'(A)$

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CNF:

- $S \rightarrow \epsilon$
- $A \rightarrow F \in \Sigma$
- $A \rightarrow BC \quad (B+C \neq S)$
- $B, C \in (V - \epsilon \Sigma^3)$

CFG:

(V, Σ, R, S)

"If the string is long enough, there's a predictable pattern"

In the game, the CFG spends variables, we force it to

Suppose $w \in A$.

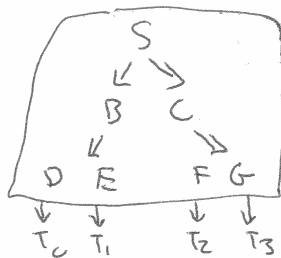
$|w| = 0$, how many vars are spent? $\perp (S)$

$|w| = 1$, $\perp (S) \quad S \rightarrow T_0$

$|w| = 2$, $S \rightarrow BC \rightarrow T_0 T_1 \quad [2, 3]$
 \searrow
 BB

$|w| = 3$, $S \rightarrow BC \rightarrow T_0 DE \rightarrow T_0 T_1 T_2 \quad [, 5]$

$|w| = 4$, $S \rightarrow BC \rightarrow DEFG \rightarrow T_0 T_1 T_2 T_3 \quad [, 7]$



no. of chars $(|w|)$
 $=$ no. of leaves
 no. of variables used
 $=$ no. of nodes in the tree

bushy tree
 $=$ a complete
 binary tree

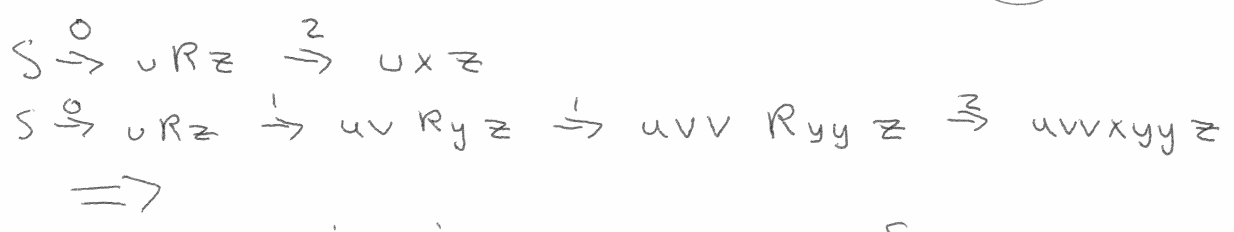
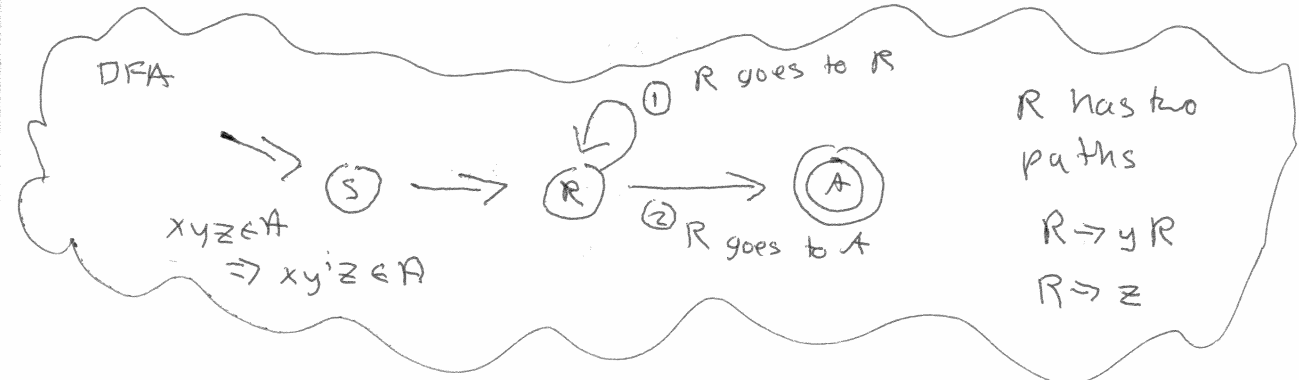
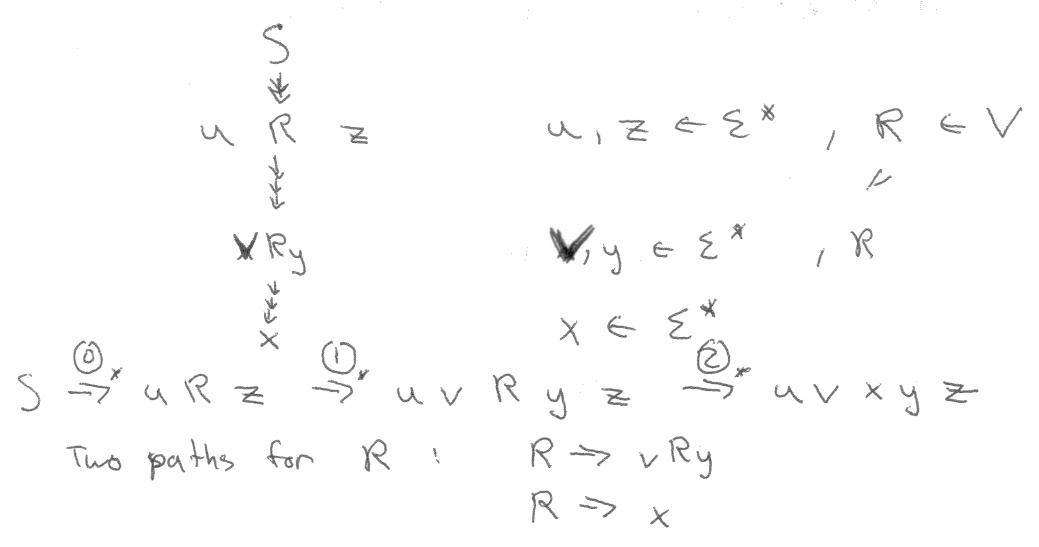
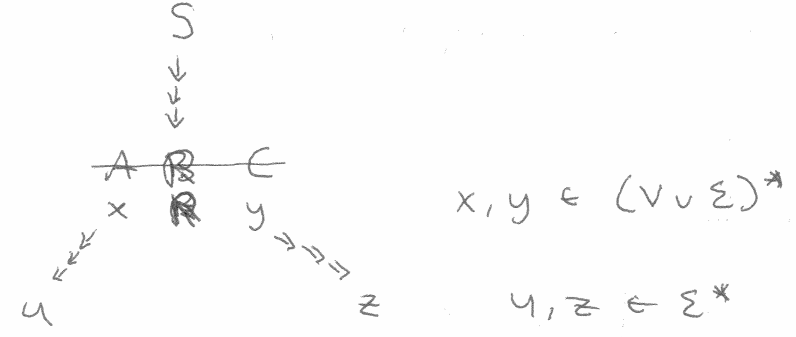
If a CBT has height H , how many nodes are there?

~~$(2^H - 1) > |V|$~~ $(2^H - 1) > |V|$

How many leaves?

$2^{(H-1)} = |w|$

$|w| = 2^{|V|} + 1$



$0^n 1^n$

$u = \epsilon \quad v = 0 \quad x = \epsilon \quad y = 1 \quad z = \epsilon$

$v^i y^i = 0^i 1^i = 0^i 1^i$

$S \rightarrow \epsilon \mid 0 S \mid 1$

$S \rightarrow 0 \mid 1 \mid S + S$

$s = 0 + 0 \quad u = \epsilon \quad v = 0^+ \quad x = \epsilon \quad y = \epsilon$

$(0^+)^i 0 \quad z = 0$



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Context-Free Pumping Property (CFPP)

$$\forall A \in \text{CFL}$$

$$\exists p \in \mathbb{N}$$

$$\forall (s \in A \mid |s| \geq p)$$

"The repeated part is not empty"

$$\exists (x, v, y, z \in \Sigma^* \mid s = uvxyz \wedge |v| > 0 \wedge |vxy| \leq p$$

"the repetition happens soon enough")

$$\forall i \in \mathbb{N}$$

$$uv^i xy^i z \in A$$

$$\neg \text{CFPP} \Rightarrow \exists A \in \text{ALL}$$

$$\forall p \in \mathbb{N}$$

$$\exists (s \in A \mid \dots)$$

$$\forall (x, v, y, z \mid \dots)$$

$$\exists i \in \mathbb{N}$$

$$uv^i xy^i z \notin A$$

$$y \notin \text{CFL} = \{ a^n b^n c^n \mid n \geq 0 \}$$