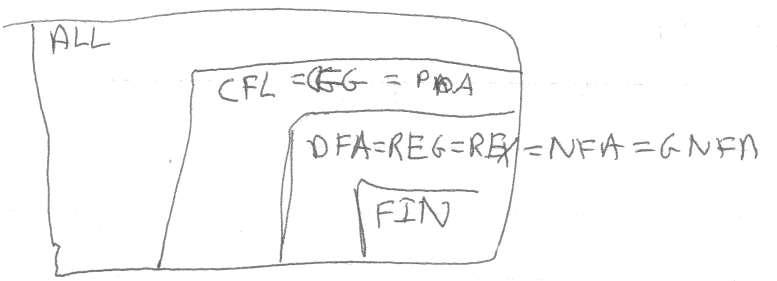
11-1/

PDA = CFG (ie PDA = ~~CFG~~ CFL)



① $\forall g \in CFG.\ \exists p \in PDA.\ L(p) = L(g)$ — compiler

② $\forall p \in PDA.\ \exists g \in CFG.\ L(g) = L(p)$ — decompiler

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Compiler

in: $g = (V, \Sigma, S, R \subseteq P(V \times (V \cup \Sigma)^*))$

assume that g is in CNF

$\forall r \in R.\ r = (S, \varepsilon)$

or (A, BC) $A,B,C \in V$ $B,C \neq S$

or (A, c) $A \in V, c \in \Sigma$

out: $p = (Q, \Sigma, \Gamma, q_0, \delta : Q \times \Sigma_\varepsilon \times \Gamma_\varepsilon \to P(Q \times \Gamma_\varepsilon), F)$

$Q = \{q_0, q_f\} \cup ??? \cup \{q_e\}$

$\Gamma = V \cup \Sigma \cup \{\$\}$

If $(S, \varepsilon) \in R$, then $\delta(q_0, \varepsilon, \varepsilon) = \{(q_f, \varepsilon)\}$

q_0

$F = \{q_f\}$

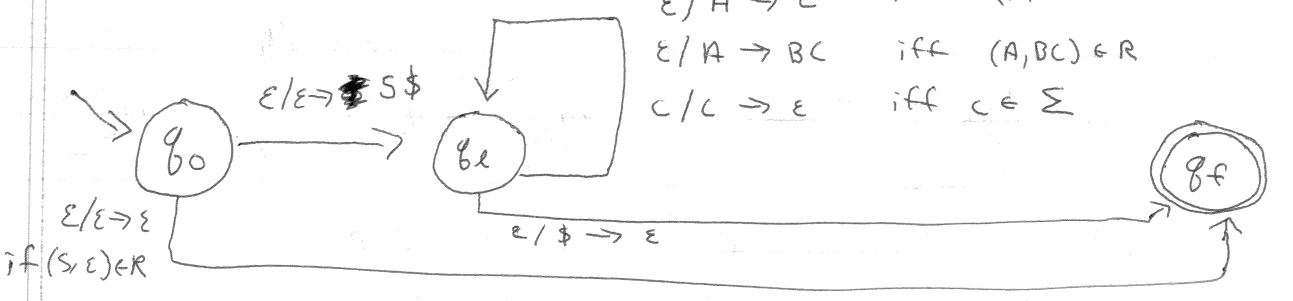
δ

$\varepsilon / S \to \varepsilon$ iff $(S, \varepsilon) \in R$

$\varepsilon / A \to C$ iff $(A, C) \in R$

$\varepsilon / A \to BC$ iff $(A, BC) \in R$

$c / c \to \varepsilon$ iff $c \in \Sigma$



$L(g) = X$ s.t. $X = F(X)$

$S \to 0S1 \mid \varepsilon$

$F : Set \to Set$

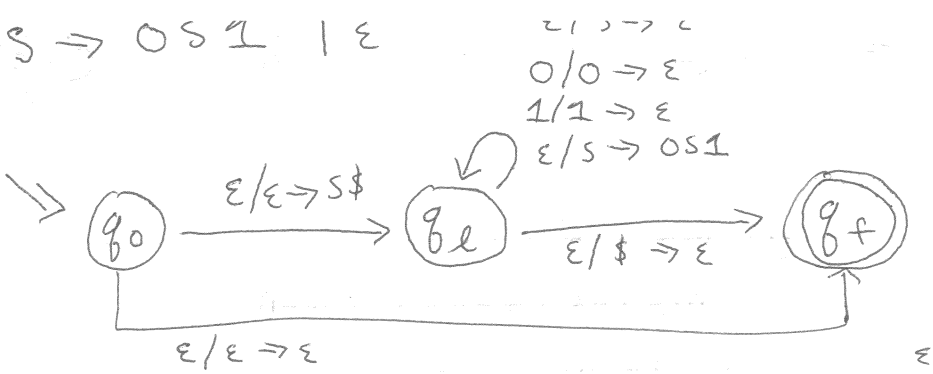
$S_0 = \varepsilon$

$F(X) = \{0x1 \mid x \in X\} \cup X$

$F(S_0) = 01, \varepsilon$ $\quad F(F(S_0)) = 0011, 01, \varepsilon$

1-2/

$S \Rightarrow 0S1 \mid \varepsilon$



000111

$q_0/\varepsilon \xrightarrow{\varepsilon} q_e/S\$ \xrightarrow{\varepsilon} q_e/0S1\$ \xrightarrow{0} q_e/S1\$$

$\downarrow \varepsilon$

$q_e/0S11\$ \xrightarrow{0} q_e/S11\$ \xrightarrow{\varepsilon} q_e/0S111\$ \xrightarrow{0} q_e/S111\$$

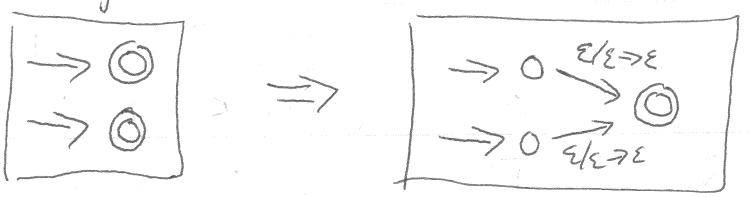
$\downarrow \varepsilon$

$q_e/111\$ \xrightarrow{1} q_e/11\$ \xrightarrow{1} q_e/1\$ \xrightarrow{1} q_e/\$ \xrightarrow{\varepsilon} q_f/\varepsilon$ ✓

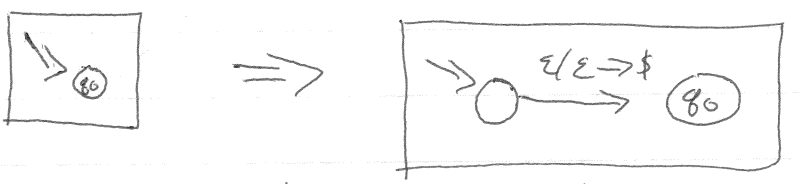
$q_e/\$ \xrightarrow{\varepsilon} q_f/\varepsilon$ ✗

Assumptions about PDA for decompiler

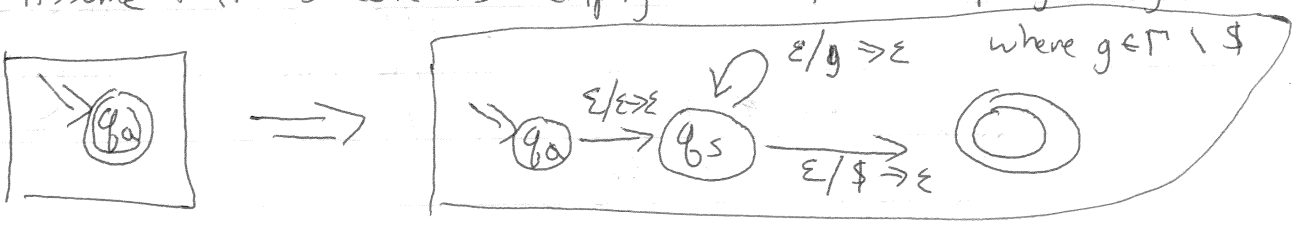
1) Single accept state



2) Assume that $ is pushed on stack first and once



3) Assume that stack is empty on all accepting strings



4) Every transistion pushes XOR pops

push : $c/\varepsilon \to g$] only allowed

pop : $c/g \to \varepsilon$

neither : $c/\varepsilon \to \varepsilon$] $\Rightarrow$ $c/\varepsilon \to g$, $\varepsilon/g \to \varepsilon$

both : $c/g_0 \to g_1$] $\Rightarrow$ $c/g_0 \to \varepsilon$, $\varepsilon/\varepsilon \to g_1$

11-3/ Idea: $\forall p, q \in Q$

$p \overset{w}{\underset{\varepsilon}{\Rightarrow}}{}^{*} q$] run machine from p to q using w w/ empty stack

iff $V_{pq} \Rightarrow^{*} w$

$V_{oo} \to \varepsilon$	V_{eo}	V_{fo}
V_{oe}	V_{ee}	V_{fe}
V_{of}	V_{ef}	V_{ff}

1 $\forall p.\ V_{pp} \to \varepsilon$

$V_{oo} \to \varepsilon$
$V_{ee} \to \varepsilon$
$V_{ff} \to \varepsilon$

2 $\forall pqr.$

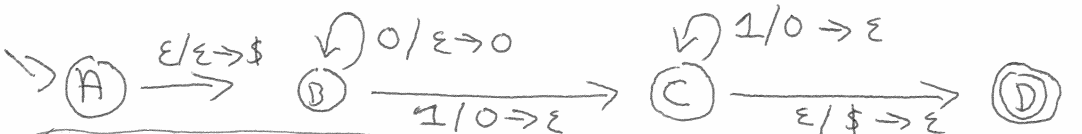
$V_{pq} \to V_{pr} V_{rq}$ $\quad V_{oo} \to V_{oo} V_{oo}$ ...
$V_{oe} \to V_{of} V_{fe}$ ...

3 $\dfrac{(r,t) \in \delta(p,a,\varepsilon) \quad (q,\varepsilon) \in \delta(s,b,t)}{V_{pq} \to a\, V_{rs}\, b}$ $\quad t \in \Gamma$ $\quad a, b \in \Sigma_\varepsilon$

$Q, \Sigma, \Gamma \Rightarrow Q, \Gamma$

δ: $A, \varepsilon, \varepsilon \to B, \$$
$C, \varepsilon\ \$ \to D, \varepsilon$

$r=B, t=\$, p=A, a=\varepsilon$
$q=D, s=C, b=\varepsilon$



$V_{AD} \to \varepsilon\ V_{BC}\ \varepsilon$

$V_{BC} \to 0\ V_{BB}\ 1$
$\quad \mid\ 0\ V_{BC}\ 1$

$V_{BB} \to \varepsilon$

$[r=B, t=0, p=B, a=0]$ $B, 0, \varepsilon \to B, 0$

case 1: $[q=C, s=B, b=1]$ $B, 1, 0 \to C, \varepsilon$

case 2: $[q=C, s=C, b=1]$ $C, 1, 0 \to C, \varepsilon$

$S = V_{q_0, q_a} = V_{AD}$