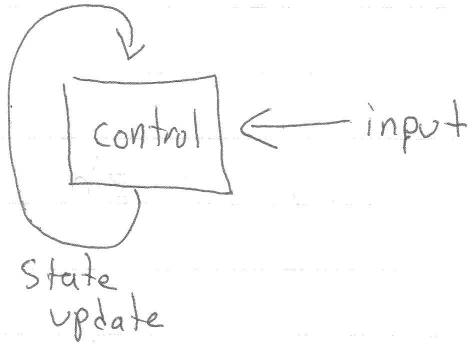
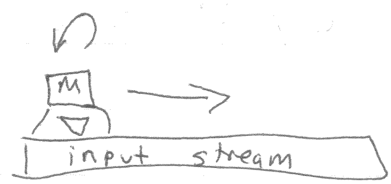
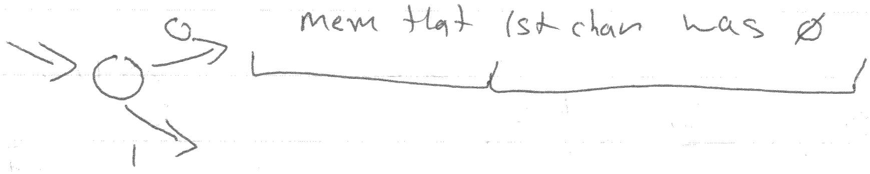
DFA : RE~~X~~ :: ____ : CFG

new computer

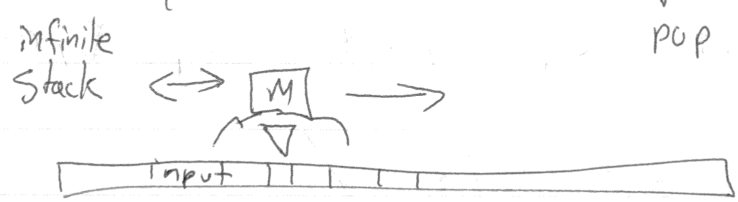
PDA — push-down automata







PDA will have unbounded-mem (DFA was bounded)
with restricted access
only stack access — push 1 thing
pop 1 thing



$$A \xrightarrow{c/d \to e} B$$

In state A if we read a 'c' and pop a 'd' then go to B and push an 'e'

$c/\varepsilon \to e$ (push e)

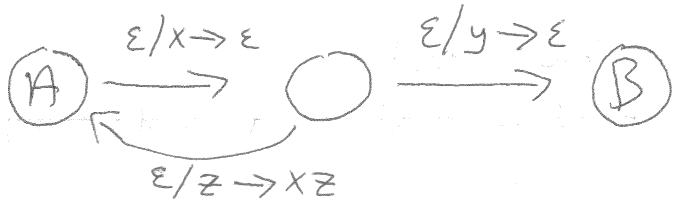
$c/d \to \varepsilon$ (pop d)

want to push x ~~and~~ y then goto B

$$A \xrightarrow{c/\varepsilon \to x} \bigcirc \xrightarrow{\varepsilon/\varepsilon \to y} B$$

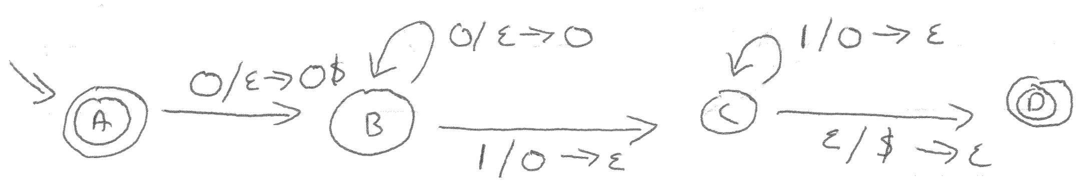
10-2

"on the stack top being x and y goto B"



$c \,/\, d^* \to e^*$ $\qquad$ $c \,/\, xy \to zh$

CFL has $0^n 1^n$ $\qquad$ $S \to 0S1 \mid \varepsilon$



000111 $\qquad$ $A/\varepsilon \xrightarrow{0} B/0\$ \xrightarrow{0} B/00\$ \xrightarrow{0} B/000\$ \xrightarrow{1} C/00\$ \xrightarrow{1} C/0\$ \xrightarrow{1} C/\$ \xrightarrow{\varepsilon} D/\varepsilon$ ✓

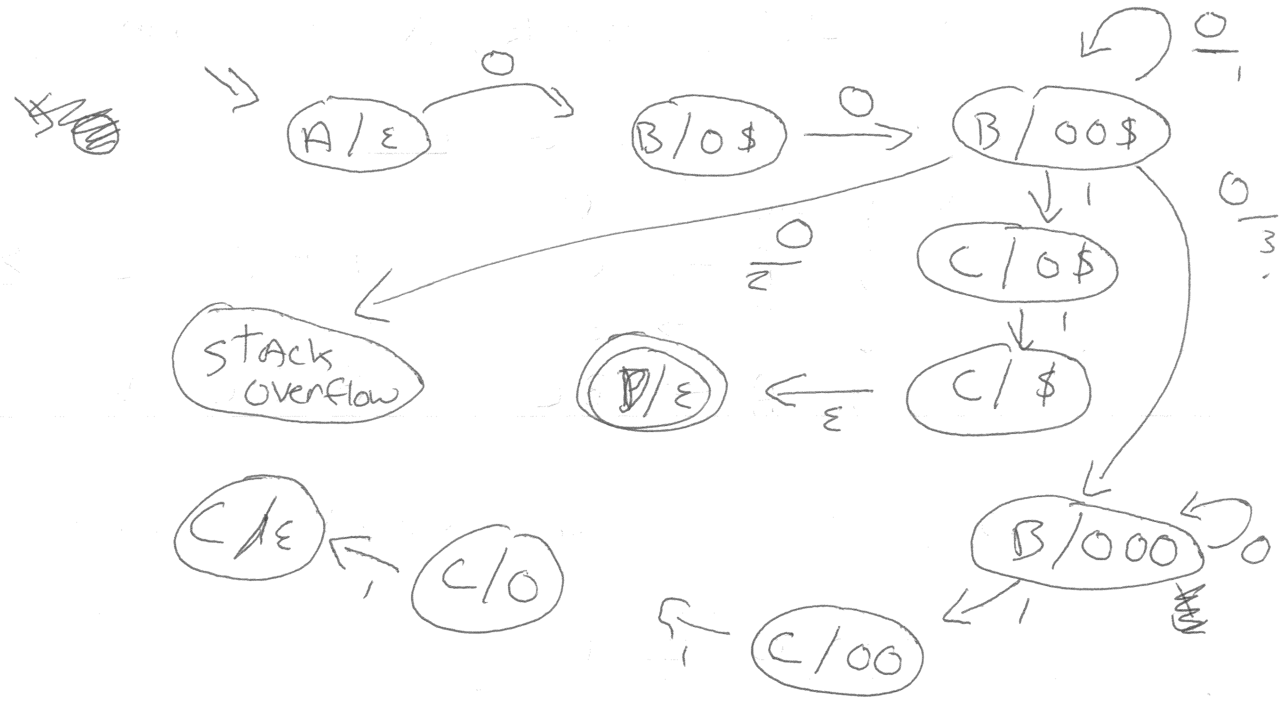
A PDA p is $Q \times \Sigma \times \Gamma \times q_0 \times \delta \times F$

- Q is a finite set of states
- Σ is an alphabet (input)
- Γ is an alphabet (stack)
- $q_0 \in Q$ is the start state
- $F \subseteq Q$ are the accepting states

$$\delta : \underbrace{Q}_{\text{from state}} \times \underbrace{\Sigma_\varepsilon}_{\text{read (may)}} \times \underbrace{\Gamma_\varepsilon}_{\text{pop (may)}} \longrightarrow \underbrace{P}_{\text{many options i.e. non-det}}(\underbrace{Q}_{\text{to state}} \times \underbrace{\Gamma_\varepsilon}_{\text{push (may)}})$$

10-3

Suppose stack is exactly 3.



The language of a PDA $p = L(p)$

$L(p) = \{ w \mid q_0 \xRightarrow[\varepsilon]{w}{}^{*} q_f \text{ where } q_f \in F \}$

q_i reaches q_k via w with stack g $\left(q_i \xRightarrow[g]{w}{}^{*} q_k \right)$

$w \in \Sigma^*$ $\quad g \in \Gamma^*$

$$q_i \xRightarrow[g]{\varepsilon}{}^{*} q_i \qquad \frac{(q_j, c) \in \delta(q_i, a, b) \qquad q_j \xRightarrow[cg]{w}{}^{*} q_k}{q_i \xRightarrow[bg]{aw}{}^{*} q_k}$$

$a \in \Sigma_\varepsilon$ $\quad w \in \Sigma^*$

$b, c \in \Gamma_\varepsilon$ $\quad g \in \Gamma^*$

10-4)

A special case of a CFG (context-free grammar) is a CNF (Chomsky Normal Form grammar)

A CNF is a CFG <u>but</u> every rule is either

1) $A \to c$ $\qquad A \in V \qquad c \in \Sigma$
2) $S \to \varepsilon$
3) $A \to BC$ $\qquad A, B, C \in V \qquad B, C \neq S$

Every CFG can be converted to CNF

0) $S \to 0S1 \mid \varepsilon$

1) $S \to ASB \mid \varepsilon$ $\qquad A \to 0 \qquad B \to 1$

2) $S \to AX \mid \varepsilon$ $\qquad A \to 0 \qquad B \to 1$
$X \to SB$

3) $S \to AX \mid \varepsilon$ $\qquad A \to 0 \qquad B \to 1$
$X \to AXB \mid B$

4) $S \to AX \mid AB \mid \varepsilon$
$X \to AXB \mid ABB$

5) $S \to AX \mid AB \mid \varepsilon$ $\qquad A \to 0 \qquad B \to 1$
$X \to AY \mid AZ$
$Y \to XB$
$Z \to BB$