Using Rewriting to Synthesize Functional Languages to Digital Circuits

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Abstract. A straightforward synthesis of functional languages to digital circuits transforms variables to wires. The type of these variables determines the bit-width of the wires. Assigning a bit-width to polymorphic and function-type variables within this direct synthesis scheme is impossible. Using a term rewrite system, polymorphic and function-type binders can be completely eliminated from a circuit description given only minor, reasonable, restrictions on the input. The presented term rewrite system is used in the compiler for CλaSH: a polymorphic, higher-order, functional hardware description language.

1 Introduction

This paper describes the use of a Term Rewriting System (TRS) in the compiler for CλaSH [1, 2]. CλaSH is a polymorphic, higher-order, functional hardware description language. The purpose of the CλaSH compiler is to transform a description in a functional language to a format from which lithography machines can build an actual chip. The CλaSH compiler actually only provides a part of this transformation. It creates a low-level representation of the hardware, called a netlist; industry-standard tools are used for further processing. The translation from a (functional) description to a netlist is called *synthesis* in hardware literature. And the set of rules/transformations that together describe the conversion from *description* to *netlist* is called a *synthesis scheme*.

The *synthesis scheme* used by the CλaSH compiler produces a specific *normal form* of the description. One aspect of this normal form is that the arguments and results of functions must have a representable type: a type whose values can be encoded by a fixed number of bits. This paper *only* describes the TRS that is used by the CλaSH compiler to eliminate, in a meaning-preserving manner, non-representable values from a functional description. The actual normal form, the TRS used for simplification, and the complete synthesis scheme, are however not described. These aspects will be described in a future paper. This paper focuses on the TRS for non-representable value elimination, because it, among other things, transforms higher-order descriptions to first-order descriptions. Because first-order programs are susceptible to a greater range of analysis techniques [3], the described TRS has value in a broader context.

The next subsection gives both a definition for netlists, and an introduction to synthesis schemes by describing a specific instance for a small functional language. The definition and introduction are both informal, but hopefully instil an intuition for the process of transforming a functional description to actual hardware.

1.1 Netlists & Synthesis

A netlist is a textual description of a digital circuit [4]. It lists the components that a circuit contains, and the connections between these components. The connection points of a component are called ports, or pins. The ports are annotated with the bit-width of the values that flow through them. A netlist can either be hierarchical or flattened. In a hierarchical netlist, sub-netlists are abstracted to appear as single components, of which multiple instances can be created. By instantiating these instances, a flattened netlist can be created.

A synthesis scheme defines the procedure that transforms a (functional) description to a netlist. Synthesis schemes defined for different languages, which nonetheless have common aspects, will be called a synthesis scheme *family*. The CλaSH compiler uses a synthesis scheme, called $\mathcal{T}_{C\lambda}$, that is an instance of the synthesis scheme family that will be referred to as $\mathcal{T}$. The following aspects are shared by all instances of $\mathcal{T}$:

- It is completely syntax-directed.
- It will create a hierarchical netlist.
- Function *definitions* are translated to components, where the arguments of the function become the input ports, and the result is connected to the output port.
- Function *application* is translated to an instance of the component that represents the corresponding function. The applied arguments are connected to the input ports of the component instance.

To demonstrate $\mathcal{T}$, a simple functional language, $\mathcal{L}$, is introduced in Fig. 1. $\mathcal{L}$ is a pure, simply-typed, first-order functional language. A program in $\mathcal{L}$ consists of set of function definitions, which always includes the *main* function. Expressions in $\mathcal{L}$ can be: variable references, primitives, or function application. Figure 2 shows a small example program defined in the presented functional language.

The synthesis scheme for $\mathcal{L}$, called $\mathcal{T}_{\mathcal{L}}$, is defined by two transformations: $[\![\ ]\!]_p$ and $[\![\ ]\!]_e$, in which $[\![\ ]\!]_p$ is initially applied to the *main* function to create the hierarchical netlist. A graphical, informal, definition of the $[\![\ ]\!]_p$ and $[\![\ ]\!]_e$ transformations is depicted in Fig. 3. Again, the purpose of this subsection is to give an intuition for the synthesis process, not to give a formal account of $\mathcal{T}_{\mathcal{L}}$. $[\![\ ]\!]_p$ creates a component definition for a function f, where input ports correspond to the argument of f. $[\![\ ]\!]_p$ also creates an output port for the result of the expression e, which is connected to the outcome of the $[\![\ ]\!]_e$ transformation applied to e.

Figure 3 shows that $[\![\ ]\!]_e$ transforms an argument reference x to a connection with an input port x. Function application of a function f is transformed to a

$$
\begin{array}{lll}
p ::= f\ \overline{x} = e;\ p & & \text{Function definitions} \\
\quad \mid\ \emptyset & & \\
 & & \\
e ::= x & & \text{Argument reference} \\
\quad \mid\ \otimes\ \overline{e} & & \text{Primitive} \\
\quad \mid\ f\ \overline{e} & & \text{Function application}
\end{array}
$$

Fig. 1. $\mathcal{L}$: a simple functional language

$$
\begin{array}{l}
double\ x\ = x * x \\
main\ x\ y = (double\ x) + (double\ y)
\end{array}
$$

Fig. 2. Example program in $\mathcal{L}$

component instance of f. $[\![\ ]\!]_p$ will be called for the definition of f in case there is no existing component definition. Arguments to f are recursively transformed by $[\![\ ]\!]_e$, and the outcome of these transformations are connected to the input ports of the component f. The process for the transformations of primitives is analogous to that of functions, except that $[\![\ ]\!]_p$ will not be called for the definitions.

1.2 Synthesis of CλaSH

Applying the synthesis scheme $\mathcal{T}_\mathcal{L}$ to the example program given in Fig. 2 results in the (graphical representation of the) netlists depicted in Fig. 4. The netlist representation of *main* shows that synthesis schemes belonging to $\mathcal{T}$ exploit the implicit parallelism present in (pure) functional languages: as there are no dependencies between the operands of the addition, they are instantiated side-by-side. During the actual operation of the circuit, electricity flows through all parts simultaneously, and the instances of *double* will actually be operating in parallel.

CλaSH is a polymorphic, higher-order, functional hardware description language. It has a syntax and a semantics similar to the programming language Haskell [5] including some of language extensions of the Glasgow Haskell Compiler (GHC) [6]. These extensions include higher-rank polymorphism and existential

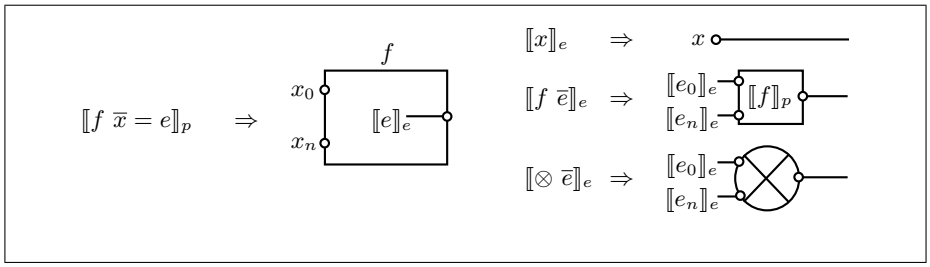


Fig. 3. $\mathcal{T}_\mathcal{L}$: A synthesis scheme for $\mathcal{L}$

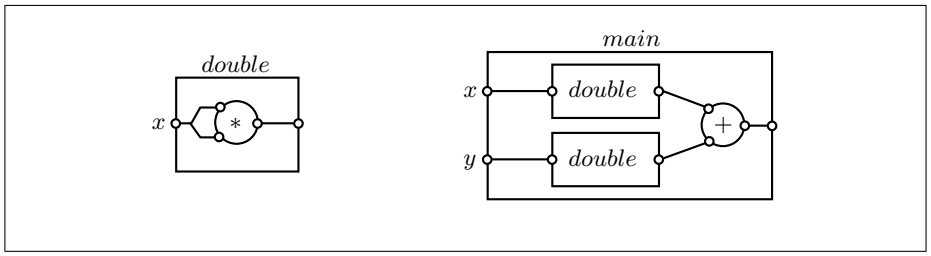


Fig. 4. Netlist of the example program in Fig. 2, created by $\mathcal{T}_{\mathcal{L}}$

datatypes. CλaSH and Haskell are so similar that every valid CλaSH description is also a valid Haskell program. Because CλaSH uses a synthesis transformation belonging to $\mathcal{T}$, called $\mathcal{T}_{C\lambda}$, the reverse relation does not hold. There are (many) Haskell programs that are not valid CλaSH descriptions. For example, a large range of recursive functions are not valid CλaSH descriptions: under $\mathcal{T}_{C\lambda}$, recursive application of a function f would lead to a recursive instance of the component *f*. Flattening the netlist would lead to infinitely many instantiations of the component *f*. Because such a netlist cannot be realized, the corresponding recursive function is currently deemed an invalid CλaSH description.

CλaSH uses an instance of the $\mathcal{T}$ family of synthesis schemes because it exploits the implicit parallelism of the functional descriptions, as shown earlier in Fig. 4. An important aspect of $\mathcal{T}$ is that the arguments and results of functions become the input and output ports of components. These ports are annotated with a bit-width so that it is known how many wires are needed to make connections between ports. Because CλaSH is a polymorphic, higher-order language, the arguments and results can however contain polymorphic or function-typed values. *It is non-trivial, if at all possible, to determine a fixed bit-width for such values.*

In order to run $\mathcal{T}_{C\lambda}$, all values that cannot be represented by a fixed bit-width, will have to be eliminated from the functional description. The focus of this paper is a TRS that transforms the functional description in a meaning-preserving manner so that all non-representable values are eliminated. This is achieved using both inlining and specialisation transformations [3].

The remainder of this paper is structured as follows: related work is described in the next section. CλaSH is desugared to a smaller Core language, and it is the Core language on which the TRS operates; Sect. 3 describes this Core language. Section 4 defines the (data)types which are considered non-representable, and the general process required for their meaning-preserving elimination. The rewrite rules and strategy of the TRS are described in Sect. 4.1 and 4.2. Properties of the TRS, including it's non-termination, and the subsequent measures taken in the CλaSH compiler are discussed in Sect. 5. Conclusions are presented in Sect. 6.

2 Related Work

Functional Hardware Description Languages. SAFL [7] is a first-order hardware description language. As opposed to $\mathcal{T}_{C\lambda}$, which is used by CλaSH,

SAFL uses a synthesis scheme that does not create a new component instance for every application of a function f. Instead, a component f is instantiated only once, and additional control and scheduling logic is inferred to safely approximate concurrent access.

BlueSpec SystemVerilog [8] is a hardware description language with a syntax similar to IEEE SystemVerilog standard. It has features also found in functional languages, such as higher-order functions and parametric polymorphism. The compilation from description to netlist is performed in two stages, which corresponds to the static and dynamic semantics of the language:

- A description is partially evaluated according to the static semantics, this includes the elimination/propagation of higher-order functions.
- The resulting description after partial evaluation is actually a set of rewrite rules. The second synthesis transformation instantiates all these rules in parallel, and adds scheduling logic in case there are conflicting preconditions [9].

So where the CλaSH compiler uses a TRS to eliminate non-representable values (such as those with a function type), the BlueSpec compiler uses a partial evaluator.

Lava [10, 11] is a domain specific language *embedded in* Haskell. A hardware description in Lava is actually a Haskell program that uses combinators made available by the Lava library. These combinators wrap constructors of a graph datatype that represents a netlist. Synthesis of Lava descriptions is not performed in the traditional sense of transforming a description to a netlist. By *running* the Lava description, a Haskell program, the complete graph representing the netlist is simply calculated/constructed. Consequently, Lava gets the synthesis of higher-order, and recursive functions, *for free*: as long as the function *calculating* the graph terminates, a netlist can be created. Being an embedded language, Lava has disadvantages compared to a compiled language such as CλaSH:

- Because a program calculating the netlist graph cannot *observe* the individual functions, there can be no intuitive function-to-component mapping. As a result, only flattened netlists can be created.
- The rich set of *choice*-constructs in Haskell (also present in CλaSH), such as pattern-matching, cannot be reflected down to the netlists. Haskell's choice construct can be used to *guide the construction* of the netlist graph, but they cannot be *observed*. Consequently, a developer using Lava only has access to *choice*-functions offered by the Lava library.

Verity is a higher-order functional hardware description language with support for recursion (using a fix-point combinator) and mutable reference-cells. Verity uses a *semantics*-directed synthesis scheme called *Geometry of Synthesis (GoS)* [12]. GoS assumes a linear type system, that restricts the use of identifiers to *exactly once*. That means that arguments with a function type need to be instantiated only once, an aspect GoS exploits during synthesis. Given a higher-order function f, which has a function-type argument g, the component

corresponding to f is given extra input and output ports. The extra output ports correspond to the input ports for g, and the extra input ports correspond to the output ports of g. When f is applied to a function h, an instance of both the f and h component are created, and the components are correctly connected to each other. CλaSH does not have a linear type-system, meaning an identifier with a function type can be applied multiple times. Because of this, the CλaSH compiler cannot use the synthesis approach for function-typed arguments as promoted by GoS.

Higher-Order removal methods. Reynolds-style defunctionalisation [13] is a well-known method for generating an equivalent first-order program from a higher-order program. Reynolds' method creates datatypes for arguments with a function-type. Instead of applying a higher-order function to a value with a function-type, it is applied to a constructor for the newly introduced datatype. Application of the functional argument is replaced by the application of a mini-interpreter. Given the following higher-order program:

$$
\begin{aligned}
&\mathit{uncurry}\ f\ (a, b) = f\ a\ b \\
&\mathit{main}\ x = (\mathit{uncurry}\ (+)\ x) + (\mathit{uncurry}\ (-)\ x)
\end{aligned}
$$

Reynolds' method creates the following behaviourally equivalent first-order program:

$$
\begin{aligned}
&\textbf{data}\ \mathit{Function} = \mathit{Plus} \mid \mathit{Sub} \\
&\mathit{apply}\ \mathit{Plus}\ a\ b = (+)\ a\ b \\
&\mathit{apply}\ \mathit{Sub}\ \ a\ b = (-)\ a\ b \\
&\mathit{uncurry}\ f\ (a, b) = \mathit{apply}\ f\ a\ b \\
&\mathit{main}\ x = (\mathit{uncurry}\ \mathit{Plus}\ x) + (\mathit{uncurry}\ \mathit{Min}\ x)
\end{aligned}
$$

Reynolds' method works on all programs, removes all functional arguments, and preserves sharing (a subject that will be discussed later). Although commonly defined and studied in the setting of the simply typed lambda calculus, there are also variants [14, 15] of Reynolds' methods that are correct within a polymorphic type system. The disadvantage of Reynolds' method is the introduction of the mini-interpreter (which takes on the form of the *apply* function in the example). Due to the parallel nature of $\mathcal{T}_{C\lambda}$, this interpreter and all of its corresponding functionality will be instantiated at the use sites of the interpreter. For the above example it means that the interpreter will be instantiated twice; and so will the included functionality: the adder and the subtracter. This method, in combination with $\mathcal{T}_{C\lambda}$, thus creates a lot of redundant hardware; it is hence not used by the CλaSH compiler.

Many of the rewrite rules used by the TRS described in this paper can also be found in optimizing compilers for functional languages, such as GHC [16]. The

TRS presented in this paper has many commonalities with the TRS presented in the work of Mitchell and Runciman [3]. The purpose of Mitchell and Runciman's TRS is transforming higher-order programs to a first-order equivalent, and is also based on a combination of specialisation and inlining. Unlike Mitchell and Runciman's TRS, the TRS presented in this paper works in a typed setting, eliminates higher-rank polymorphic arguments, and eliminates existential datatypes.

3 Core Language

The syntactically rich CλaSH language is desugared to a smaller Core language, called Core_{HW} (Fig. 5), by the CλaSH compiler. It is a Church-style polymorphic lambda-calculus extended with primitives, algebraic datatypes in combination with case-decomposition, and recursive let-bindings. Case-decompositions are either exhaustive in the constructors of the matched datatype, or include the default pattern. Figure 5 gives the language definition of Core_{HW}, and uses, just like the rest of this paper, the notation described in Fig. 6.

Local variables: x, y, z	**Data Constructors:**
Global Variables: f, g	$K : \forall\overline{\alpha}.\forall\overline{\beta}.\overline{\tau} \rightarrow \mathbf{T}\ \overline{\alpha}$
Type Variables: α, β	
Types	
$\tau, \sigma ::= \alpha$	Type variable references
$\mid\ \tau \rightarrow \sigma$	Function Types
$\mid\ \mathbf{T}$	Datatype Constructors
$\mid\ \tau\ \sigma$	Type application
$\mid\ \forall\alpha.\sigma$	Polymorphic types
Expressions	
$e, u ::= x \mid f$	Variable reference
$\mid\ K \mid \otimes$	Data Constructor / Primitive Function
$\mid\ \Lambda\alpha.e \mid e\ \tau$	Type abstraction / application
$\mid\ \lambda x : \sigma.e \mid e\ u$	Term abstraction / application
$\mid$ **let** $\overline{x : \sigma = e}$ **in** u	Recursive let-binding
$\mid$ **case** e **of** $\overline{p \rightarrow u}$	Case decomposition
Patterns	
$p ::= \_$	Default case
$\mid\ K\ \overline{\beta}\ \overline{x : \sigma}$	Match data constructor

Fig. 5. The Core_{HW} calculus

CλaSH supports existential datatypes, and this aspect of the language is reflected in Core_{HW}. A data constructor K, for an existential datatype $\mathbf{T}\ \overline{\alpha}$, is first abstracted over the universally quantified type-variables $\overline{\alpha}$, followed by the existentially quantified type-variables $\overline{\beta}$. The type variables $\overline{\beta}$ brought into scope by a pattern in a case-decomposition correspond to the existentially-quantified type-variables of the datatype.

$$
\begin{array}{lll}
\mathbf{T}\ \overline{\sigma} & \equiv \mathbf{T}\ \sigma_1\ ...\ \sigma_n & e\ \overline{u} \quad\ \equiv e\ u_1\ ...\ u_n \\
\overline{\tau} \to \sigma & \equiv \tau_1 \to ... \to \tau_n \to \sigma & \lambda\overline{x:\sigma}.e \equiv \lambda x_1 : \sigma_1.\ ...\ \lambda x_n : \sigma_n.e \\
\forall\overline{\alpha}.\sigma & \equiv \forall\alpha_1.\ ...\ \forall\alpha_n.\sigma & \overline{x:\sigma=e} \equiv \{x_1 : \sigma_1 = e_1,\ ...\ ,\ x_n : \sigma_n = e_n\} \\
 & & \overline{p \to u} \quad\ \equiv \{p_1 \to u_1,\ ...\ ,\ p_n \to u_n\}
\end{array}
$$

$$
K\ \overline{\beta}\ \overline{x:\sigma}\ \equiv\ K\ \beta_1\ ...\ \beta_n\ (x_1 : \sigma_1)\ ...\ (x_m : \sigma_m)
$$

Fig. 6. Notation

3.1 Synthesis of Core_{HW} using $\mathcal{T}_{C\lambda}$

The synthesis scheme $\mathcal{T}_{C\lambda}$ exploits all the implicit parallelism available in the Core_{HW} language. It does this by instantiating all expressions in a let-binding, and all alternatives of a case-decomposition side-by-side (Fig. 7). $\mathcal{T}_{C\lambda}$ creates anchor points for let-bindings so that variable references can be synthesized to connections to these anchor points.

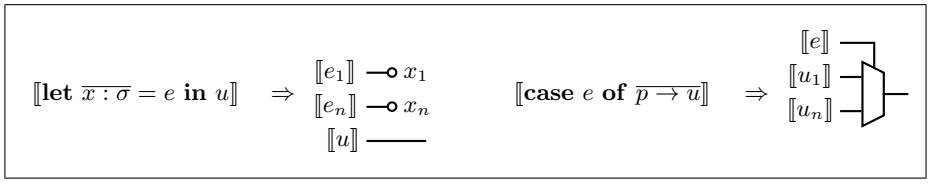


Fig. 7. Synthesis of **let** and **case**

```
λx : Bool.λy : Int.let
  z : Int = y ∗ y
in case x of
    True → z + 1
    False → z − 1
```

Fig. 8. Example program using **let** and **case**

Completely elaborating $\mathcal{T}_{C\lambda}$ falls outside of the scope of this paper. To at least convey an intuition for the synthesis performed by $\mathcal{T}_{C\lambda}$, an example program, and the corresponding netlist are shown in Fig. 8 and 9. The simultaneous presence of all alternatives in a case decomposition, and all bindings in let-binding, has consequences for the *sharing* behaviour of expressions.

Sharing is normally defined as the *re-use of the result of a computation* by other expressions. In a digital circuit, sharing means connecting the output port of one component to the input ports of multiple other components. This aspect can be observed in Fig. 9, where the result of the multiplication is shared by the addition and the subtraction. Results that can be shared, instead of recomputed, will reduce the total size of the circuit. The rewrite rules of the TRS should

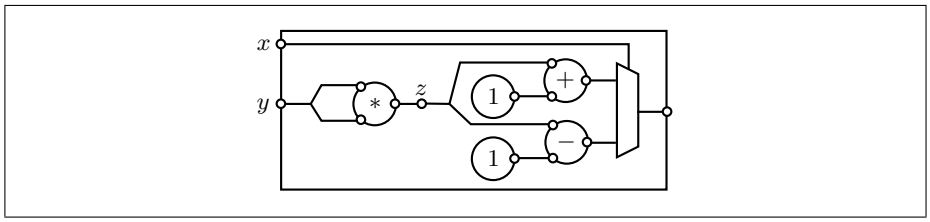


Fig. 9. Netlist of the example program in Fig. 8, created by $\mathcal{T}_{C\lambda}$

thus take the effects of sharing under $\mathcal{T}_{C\lambda}$ into account, as any loss in sharing increases the size of the circuit.

4 Eliminating Non-representable Types

$\mathcal{T}_{C\lambda}$ can only synthesize functional descriptions if arguments and results of expressions can be given a fixed bit-encoding. There are straightforward encodings for certain primitive datatypes, and certain algebraic datatypes. Deriving a fixed bit-encoding for the following types is either not desired, or not possible:

- Function types
- (Higher-rank) polymorphic types
- Datatypes that are composed of the above, non-representable, types.

Given the following circumstances:

- All arguments and the result of the *main* function are representable.
- All arguments and results of primitives are representable.

a combination of inlining and specialisation (and dead-code elimination) can completely eliminate non-representable values from the function hierarchy. Specialisation in this case takes on two forms:

- Specialisation of a function on one of its arguments.
- Elimination of a case-decomposition based on a known constructor.

The next two subsection describe the rewrite rules and strategy for a TRS that achieves the specified specialisation and inlining.

4.1 Rewrite rules

The rewrite rules in this paper are presented using the format depicted in Fig. 10. In all of these rewrite rules, the expression above the horizontal bar is the expression that has to be matched before performing the rewrite rule, and the expression below the horizontal bar is the result after applying the rewrite rule. Some rewrite rules have additional preconditions, and the rewrite is only applied when these preconditions hold. Other rewrite rules have additional definitions which they use in the resulting expressions. All rewrite rules always have access

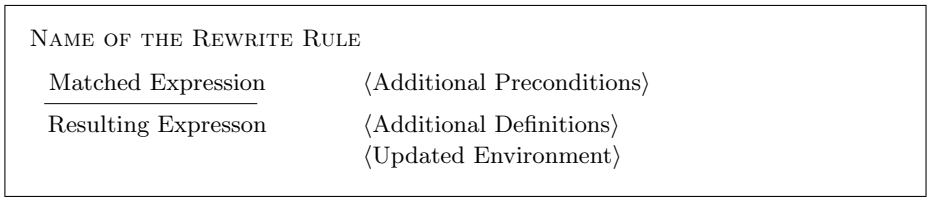


Fig. 10. Format for Rewrite Rules

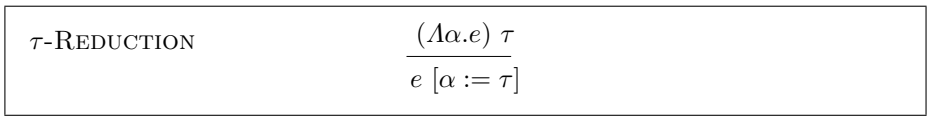
to the global environment, Γ, which holds all function bindings. There are some rewrite rules that create new top-level binders, and therefore update the global environment.

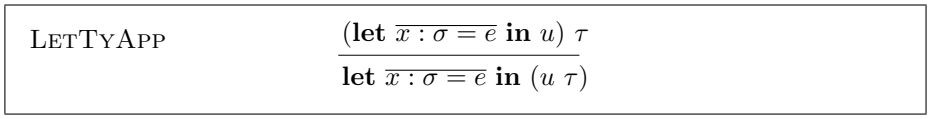
The rewrite rules have access to the following functions:

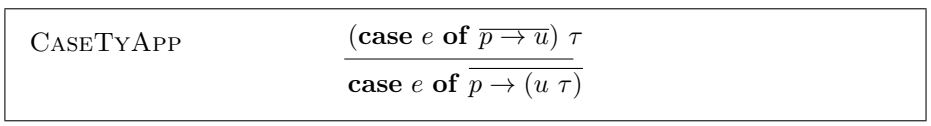
FV e	Calculates the free variables; works for types and terms.
$e\ [x := u]$	A capture-free substitution of a variable reference x, by the expression or type u, in the expression e.
$\Gamma@f$	The expression e belonging to a global binder f in the environment Γ.
NONREP τ	Determines if τ is a non-representable type.

Before the TRS starts, all variables are made unique, and all variable references are updated accordingly. Any new variables introduced by the rewrite rules will be unique by construction. Having hygienic expressions prevents accidental free-variable capture, and makes it easier to define meaning-preserving rewrite rules.

The first three rewrite rules, τ-reduction, LetTyApp, and CaseTyApp, propagate type information downwards into an expression. By either removing type-variables, propagating type-information to a location for specialisation, or propagating type information to a primitive or constructor, these rewrite rules aid in the elimination of polymorphism.







The next three rewrite rules, LamApp, LetApp, and CaseApp, propagate values, including non-representable ones, downwards into the expression. LamApp

is preferred over β-reduction to preserve sharing. CaseApp introduces a let-binding, instead of propagating the expression towards all alternatives, to preserve sharing. The next two rewrite rules, BindNonRep and LiftNonRep, remove the let-bindings introduced by LamApp and CaseApp in case they bind non-representable values.

LamApp

$$\frac{(\lambda x : \sigma . e)\ u}{\textbf{let}\ \{x = u\}\ \textbf{in}\ e}$$

LetApp

$$\frac{(\textbf{let}\ \overline{x : \sigma = e}\ \textbf{in}\ u)\ e_0}{\textbf{let}\ \overline{x : \sigma = e}\ \textbf{in}\ (u\ e_0)}$$

CaseApp

$$\frac{(\textbf{case}\ e\ \textbf{of}\ \overline{p \to u})\ u_0}{\textbf{let}\ \{x = u_0\}\ \textbf{in}\ (\textbf{case}\ e\ \textbf{of}\ \overline{p \to (u\ x)})}$$

The BindNonRep rewrite rule removes let-bindings with non-representable types that are not self-referencing (recursive), and substitutes the bound expressions in the rest of the let-expression. LiftNonRep removes a self-referencing let-binding with a non-representable type, and substitutes the binder in the rest of the let-expression with an application of a newly created global binder. The new global binder binds the original expression, abstracted over its free local (type) variables; all self-references are replaced by references to the new global binder. Substituting the reference by the bound expression, such as done in BindNonRep, is unsound for a self-referencing binder. A removal and substitution of the binder would in this case create a free variable in all substitutions.

BindNonRep

$$\frac{\textbf{let}\ \{b_1; ...; b_{i-1}; x_i : \sigma_i = e_i; b_{i+1}; ...; b_n\}\ \textbf{in}\ u}{(\textbf{let}\ \{b_1; ...; b_{i-1}; b_{i+1}; ...; b_n\}\ \textbf{in}\ u)\ [x_i := e_i]}$$

Preconditions: $\text{NONREP}(\sigma_i)$ $\wedge\ x_i \notin \text{FV}(e_i)$

LiftNonRep

$$\frac{\textbf{let}\ \{b_1; ...; b_{i-1}; x_i : \sigma_i = e_i; b_{i+1}; ...; b_n\}\ \textbf{in}\ u}{(\textbf{let}\ \{b_1; ...; b_{i-1}; b_{i+1}; ...; b_n\}\ \textbf{in}\ u)\ [x := f\ \overline{\alpha}\ \overline{z}]}$$

Preconditions: $\text{NONREP}(\sigma_i)$ $\wedge\ x \in \text{FV}(e_i)$

Definitions: $(\overline{\alpha}, \overline{y}) = \text{FV}(e_i)$; $\overline{z} = \overline{y} - \{x_i\}$

New Environment: $\Gamma \cup \{(f, \Lambda\overline{\alpha}.\lambda\overline{z}.e_i[x_i := f\ \overline{\alpha}\ \overline{z}])\}$

The previous rewrite rules either propagated non-representable values downwards into the expression, or lifted those values out of the expression. The next two sets of rewrite rules remove non-representable values by specialisation. The TypeSpec and NonRepSpec provide function argument specialisation. CaseLet, CaseCase, InlineNonRep, and CaseCon, together achieve specialisation by eliminating case-decompositions of known constructors (of non-representable datatypes).

The TypeSpec rewrite rule matches a type application of global variable reference, f. The application is replaced by a reference to the newly created global function f'. The new binder f' is defined in terms of the body of f specialized on the type τ. TypeSpec uses the $\cup_s$ operator to indicate that the global environment is only updated with a new binder if the specialization of f on τ has not been seen before. In case the specialisation has been seen before, the previously created f' variable is used in the new expression.

The NonRepSpec rewrite rule uses the $\cup_\alpha$ operator to indicate that the global environment is only updated with a new binder if an α-equivalent specialization has not been seen before. In case the specialisation has been seen before, the previously created f' variable is used in the new expression.

TypeSpec
$$\frac{(f\ \overline{e})\ \tau}{f'\ \overline{e}} \qquad \text{Preconditions: } \mathrm{FV}(\tau) \equiv \emptyset$$
$$\text{New Environment: } \Gamma \cup_s \{(f', \lambda\overline{x}.(\Gamma@f)\ \overline{x}\ \tau)\}$$

NonRepSpec
$$\frac{(f\ \overline{e})\ (u{:}\sigma)}{f'\ \overline{e}\ \overline{y}} \qquad \text{Preconditions: } \mathrm{NONREP}(\sigma) \wedge \mathrm{FV}(\sigma) \equiv \emptyset \qquad \text{Definitions: } \overline{y} = \mathrm{FV}\ u$$
$$\text{New Environment: } \Gamma \cup_\alpha \{(f', \lambda\overline{x}.\lambda\overline{y}.(\Gamma@f)\ \overline{x}\ u)\}$$

The CaseLet is required in specialising expressions that have a non-representable datatype. Taking the let-binders out of the case decomposition does not affect the sharing behaviour so can be applied blindly. There is no free variable capture in the alternatives because all variables are made unique before running the TRS.

The CaseCase rewrite rule is only applied if the subject of a case decomposition has a non-representable datatype. CaseCase is not applied blindly because the alternatives in a case-decomposition are evaluated in parallel in the eventual circuit. So the CaseCase rewrite rule generates a larger number of alternatives than present in the matched expression. A larger number of alternatives means a larger circuit. Even though CaseCase makes the circuit larger, the intention of CaseCase is to eventually expose the constructor of the non-representable datatype to CaseCon. CaseCon eliminates the case-decomposition, and subsequently amortizes the increase in circuit size induced by CaseCase.

InlineNonRep is only applied if the subject of a case expression is of a non-representable datatype, as inlining breaks down the component hierarchy. All bound variables in the inlined expression are regenerated, and variable references updated accordingly. This preserves the assumptions made by the other rewrite rules that all variables are unique.

The CaseCon rule comes in three variants:

- A case-decomposition with a constructor application as the subject, and a matching constructor pattern.
- A case-decomposition with a constructor application as the subject, with no matching constructor pattern.
- A case-decomposition with a single alternative, where the corresponding pattern is the default pattern.

CaseCon only creates a let-binding if the constructor in the subject exactly matches the constructor of an alternative. When the default pattern is matched, the case decomposition is simply replaced by the expression belonging to the default alternative. Case decompositions in Core_{HW} are exhaustive, either by enumerating all the constructors, or by including the default pattern. This means that when a constructor applications is the subject of a case-decomposition, CaseCon removes that case-decomposition.

CaseLet

$$\frac{\textbf{case}\ (\textbf{let}\ \overline{x : \sigma = e}\ \textbf{in}\ e_1)\ \textbf{of}\ \overline{p \to u}}{\textbf{let}\ \overline{x : \sigma = e}\ \textbf{in}\ (\textbf{case}\ e_1\ \textbf{of}\ \overline{p \to u})}$$

CaseCase — Preconditions: NONREP(σ)

$$\frac{\textbf{case}\ (\textbf{case}\ e\ \textbf{of}\ \{p_1 \to u_1;\ \ldots\ ;\ p_n \to u_n\} : \sigma)\ \textbf{of}\ \overline{p \to u}}{\textbf{case}\ e\ \textbf{of}\{p_1 \to \textbf{case}\ u_1\ \textbf{of}\ \overline{p \to u};\ \ldots\ ;\ p_n \to \textbf{case}\ u_n\ \textbf{of}\ \overline{p \to u}\}}$$

InlineNonRep — Preconditions: NONREP(σ)

$$\frac{\textbf{case}\ (f\ \overline{e}) : \sigma\ \textbf{of}\ \overline{p \to u}}{\textbf{case}\ ((\Gamma @ f)\ \overline{e})\ \textbf{of}\ \overline{p \to u}}$$

CaseCon

$$\frac{\textbf{case}\ K_i\ \overline{\tau_\forall}\ \overline{\tau_\exists}\ \overline{y}\ \textbf{of}\ \{\ldots; K_i\ \overline{\beta}\ \overline{x : \sigma} \to e; \ldots\}}{(\textbf{let}\ \overline{x : \sigma = y}\ \textbf{in}\ e)\ [\overline{\beta} := \overline{\tau_\exists}]} \quad \frac{\textbf{case}\ u\ \textbf{of}\ \{\_ \to e\}}{e}$$

$$\frac{\textbf{case}\ K_i\ \overline{\tau_\forall}\ \overline{\tau_\exists}\ \overline{y}\ \textbf{of}\ \{\overline{p_{j \neq i} \to u}; \_ \to e\}}{e}$$

4.2 Strategy

All rewrite rules, except TypeSpec, NonRepSpec, and InlineNonRep, are exhaustively applied using a top-down traversal on an expression. InlineNonRep is applied using a bottom-up traversal, because a top-down traversal could lead to non-termination when inlining a recursive function. After all the rewrite rules, with the exception of TypeSpec and NonRepSpec have been applied exhaustively, TypeSpec is applied using a bottom-up traversal. This is followed by a bottom-up traversal with the NonRepSpec rewrite rule. Bottom-up traversals are used so that the fewest number of lambda-abstraction is introduced in the specialized expressions. The argument-specialisation rewrite rules are applied last, so that the fewest number of new functions is introduced, and the original function hierarchy is preserved as much as possible. Because TypeSpec and NonRepSpec do not create expressions on which the other rewrite rules match, all rewrite rules have been applied exhaustively after the traversal with NonRepSpec.

5 Discussion

5.1 Completeness

The first set of rewrite rules (τ-Reduction - LiftNonRep) propagates or removes non-representable values for those syntactical elements on which the specialisation rewrite rules do not match. The second set of rewrite rules (TypeSpec - CaseCon) remove the non-representable values through specialisation. All rewrite rules together hence remove all non-representable values from the function hierarchy, given the restrictions in Section 4.

The restrictions on primitives are needed because those cannot be specialized on their argument. The restriction that the result type of *main* cannot be a non-representable datatype, ensures that either:

- An expression calculating a non-representable datatype is always the subject of a case-decomposition, which will be removed by the TRS.
- An expression calculating a non-representable datatype is unreachable, and can be removed by dead-code elimination.

5.2 Termination

There are several (combinations) of rewrite rules that induce non-termination of the unconstrained TRS. This subsection highlights those rewrite rules, and discusses what measures are taken in the CλaSH compiler to prevent non-termination. When one of the termination measures is triggered, non-representable values remain present in the description. $\mathcal{T}_{C\lambda}$ will not be able to transform the description to a netlist when that happens.

InlineNonRep Although the precondition of InlineNonRep already limits the locations where inlining is applied, exhaustive application of this rewrite rule can still induce non-termination when dealing with recursive functions. To prevent this from happening, a function *f*, can only be inlined *once* at all use sites within a function *g*, for every pair of *f* and *g*.

β-Reduction Although the TRS does not contain β-reduction as one of the rewrite rules, LamApp, CaseCon, and BindNonRep together behave like β-reduction. This means that the typed version of $(\lambda x.x\ x)\ (\lambda x.x\ x)$ induces non-termination. To ensure termination, BindNonRep is only applied n number of times within a function body, where n can be set by the user of the CλaSH compiler. When a function body is changed by InlineNonRep, the counter is reset. The limitation on the number of applications of BindNonRep is not a real problem in practice: the number of non-representable let-bindings in the average expression is usually limited.

NonRepSpec Specialization performed by NonRepSpec can induce non-termination when a recursive function f has an argument that accumulates non-representable values. To ensure termination, a NonRepSpec is only applied to a function m number of times, where m can be set by the user of the CλaSH compiler. This restriction has not posed any problems in practice.

6 Conclusions

The CλaSH compiler uses a synthesis scheme, $\mathcal{T}_{C\lambda}$, that produces a description that has specific normal from. One aspect of this normal form is that arguments and results of expressions do not have a non-representable value. For $\mathcal{T}_{C\lambda}$, non-representable values are those values for which no fixed bit-encoding can be determined. The TRS presented in this paper removes all non-representable values from a function hierarchy while preserving the behaviour, given only minor restrictions on this function hierarchy. These restrictions are: that neither the *main* function nor the primitives of CλaSH, can have arguments or results of a non-representable type. These restrictions do however not limit the use polymorphism or higher-order functionality in the rest of the description. We, the authors of this paper, deem these restrictions reasonable for the application domain of CλaSH: building hardware.

The (ideas behind the) presented TRS can also be applied in a broader context. One example is the application of first-order analysis techniques on higher-order programs [3]. Because function-type values are non-representable, the TRS transforms a higher-order program to a behaviourally equivalent first-order program. Any first-order analysis technique can be subsequently applied to the transformed program.

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